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Pandemic contagion to financial contagion model

Modelo de contagio de pandemia a contagio financiero

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Abstract

The means of transmission of contagion in infectious diseases and the financial crises are very similar. Within the possible learning than can be had from this period of COVID-19 in this work the concepts and ideas of health pandemics are used in order to model financial crises and as their origin factor, as was the case with COVID-19 in 2020. This paper present both a deterministic model and a Montecarlo simulation model in order to explained the transmission from the health pandemic to the financial pandemic. The graphical behavior is very similar to that observed in stylized events, especially in the short term.

JEL Code: C02, C63, E37, G01
Keywords: COVID-19; contagion; financial crisis; infectious diseases

Resumen

Los medios de transmisión de contagios en enfermedades infecciosas y los que provocan las crisis financieras actúan de forma muy similar. Dentro del posible aprendizaje que se puede tener de este periodo de COVID-19 en el presente trabajo se utilizan los conceptos e ideas de las pandemias de salud con el objetivo de modelar las crisis financieras y como las primeras pueden dar originen a las segundas como en el caso del COVID-19 en el 2020. Con un modelo determinista y uno de simulación Montecarlo se propone explicar la transmisión de la pandemia de salud a la pandemia financiera. El comportamiento gráfico es muy similar a lo observado en los hechos estilizados especialmente en el corto plazo.

Código JEL: C02, C63, E37, G01
Palabras clave: COVID-19; contagio; crisis financieras; enfermedades infecciosas

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Introduction

The study of contagion in financial markets and its effect on price growth and the formation of bubbles with the consequent financial crashes or crises has been the subject of countless articles and books and has been analyzed from various points of view, some of which have become classic works.

The issue of contagion is an important and widely studied topic in the literature. There are many systems where this behavior exists, ranging from contagious diseases, such as influenza, dengue, AIDS and, more recently, the COVID-19 pandemic. In the financial markets, this type of contagion behavior also exists, leading to the formation of bubbles and subsequent financial crises. Similarly, in nature there exists the behavior of contagion or imitation in the actions of animals and in societies where there are contagion processes, for example, in demonstrations, migrations, and other social behaviors.

In the case of infectious diseases, these are transmitted by a virus or bacteria, and the transmission is by person-to-person contact. For financial contagion, the contagion behavior occurs through electronic media, in other words, through e-mails, Twitter, Facebook, or browser searches based on a triggering event that is an indicator of the growth or fall of prices.

The outbreak of the COVID-19 pandemic in the city of Wuhan in China at the end of 2019 and its spread throughout the world resulted in many infections and deaths. Countries worldwide have established lockdown measures to reduce the spread of the disease and the number of deaths, but with the collateral effect of applying a brake on the economy.

The immediate and short-term impact on the economy and financial markets has been brutal worldwide, particularly for Mexico. The effects range from falling gross domestic product (GDP), rising unemployment, depreciating currencies, falling stock markets, and a general plunge in various financial assets. COVID-19 has not only impacted the real sector of the economy and the financial markets, but it has also affected production chains, worker mobility, tourism, transportation, and almost all industries, putting most economies in a crisis rarely seen before.

According to Li (2018), the objective in designing a mathematical model of infectious disease is to describe the mechanism of disease transmission, which is that infected individuals are introduced to a susceptible population, and the disease is transmitted to other individuals, spreading within the population. An individual may remain asymptomatic during early infection and gradually develop symptoms until diagnosed with this disease.

If the number of cases rises above the average over a short period, an outbreak of the disease occurs. When the disease spreads rapidly to many people, it is called an epidemic. Individuals who recover from infection by treatment or immune system response gain a certain level of immunity against reinfection (which occurs in almost all known diseases). When the pool of susceptible individuals is reached, the number of infected individuals will be reduced, and the epidemic will gradually be halted. If, in addition, several sensitive individuals are added to the population by births, migrations, or reinfections in the population, the epidemic can persist and remain in the population for long periods; in this case, it is said to be endemic to the population, and if the disease spreads to several countries and continents then a pandemic occurs.

On the other hand, according to Porras (2017), in a financial contagion it is important to understand that the formation, transmission, and implosion of a financial bubble and the consequent crises are related to social learning. In these processes, market agents observe the actions of others and ignore this information, thinking that others possess greater knowledge or intelligence, for example, when following financial gurus. However, sometimes ignoring information, submitting to the herd effect, and following the crowd by imitation has to do with the penalty for moving away from the crowd's actions if wrong.

The author mentioned above has written two books dedicated exclusively to contagion. Her publications are motivated by an interest in understanding aspects of the daily interactions between different types of investors in incomplete markets where psychological bias and other frictions occur in asset valuation and financial trading.

Bubbles are often seen in the first phase of their life and boost the affected sectors and the economy. These bubbles are an intrinsic element in the dynamics of financial markets. They are born at any opportunity or can be created with the interest of a specific group. Contagion is the mechanism that fuels the growth and transmission of certain events within various segments of the economy and among different geographical areas.

Kirman's model (1991) considers three important aspects in the development of a financial pandemic. The first is that agents make their choices according to their preferences or expectations, and the second is that agents recruit others to their specific options, convincing them that they possess superior information to that which is available to them and that this could be the result of an externality. The third aspect is that change will occur after following a regime for some time. These changes may be the outcome of an evolutionary process in which convergence is achieved after the success of a strategy leads to a self-conversion of the agents.

Sornette (2003) presents a history of financial crises and how they can be understood using the most modern and sophisticated scientific concepts. In other words, complex systems and critical phenomena, at the same level as financial market crises, provide an excuse to explore the world of self-organizing systems.

According to Sornette, market crises are fascinating moments for academics and practical people. According to the academic world, financial markets are efficient only in re-evaluating certain

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information that can cause crises. Fear of these crises is a perpetual source of stress, rather than the event itself and always ruins the lives of others. The author proposes a radically different view of the concept of crisis, that it fundamentally has an endogenous or internal origin and that external factors only serve to dissipate it. One consequence is that the roots of crises are much more subtle than one might think and develop progressively in the market as a whole or as a self-organized process. In this sense, the real cause of the crisis can be called systemic instability.

Systemic instability is a major concern of governments, central banks, and regulatory agencies, as the globalization of information and technology has led the economy to adopt new rules. With the support of the Basel Committee on Banking Supervision, it is advised that in the handling of systematic issues, adequate risk management is necessary to achieve confidence in the financial system and possible contagion. On the other hand, it is essential to minimize the distortion of market signals and discipline.

The objective of this paper is to use an existing pandemic model (SIR) to model the transmission and contagion behavior in financial markets with the initial hypothesis that the pandemic model can be extended to a financial contagion model employing a differential equation model and a Monte Carlo simulation model. The article is structured as follows: in the present section, there is an introduction to the objectives of the article, as well as the background of pandemics of disease and financial contagion. The second section reviews the literature related to the COVID-19 pandemic and the crises originated by the contagions in the financial environment and their economic effects. The third section summarizes the stylized facts. Subsequently, section four presents the models followed by the proposed models in section five, and finally, the results and conclusions.

Literature review

The subject of disease pandemics and contagion in the financial world and its models is very broad and varied, and some of the most relevant works consulted are mentioned below.

Mohsin et al. (2020) mention that the coronavirus epidemic has rapidly threatened health systems and has undoubtedly affected financial markets, research the reaction of global markets in terms of their decline and corresponding volatility as the virus moved from China to Europe and the United States, and suggest that in the dispersal stage the effect has been even more devastating.

Gai and Kapadia (2010) present a model of financial contagion that explores how an idiosyncratic shock influences the probability and potential impact of financial contagion by causing changes in the structure of the liquidity network on the asset market. It is argued that the financial system does not tend to fragility as long as the probability of contagion is low. Still, the effect can be profound when a pandemic occurs, explaining the system's resilience to large shocks such as the one in 2007.

In the paper by Okorie and Lin (2021), the effect from the point of view of a fractal contagion of COVID-19 on financial markets is researched. An ex-ante and ex-post sample of the COVID-19 outbreak in 32 affected economies is analyzed, and mobile and fixed cross-correlation analyses are compared. A fractal contagion effect of the COVID-19 pandemic in equity markets is confirmed, and its fractal contagion effects fade over time for both market returns and volatility.

In Fan and Huang (2017), in contrast to typical macroeconomic analyses of financial crises, the relation between the effects of negative messaging and the economic deterioration of the situation over time is researched. The theory of financial crises generated by the dispersion of investor panic is analyzed analogously to an infectious disease. The number of reproductions is essential to determine the propagation of negative messages in a group of investors, the maximum number of investors that can be influenced by a message, and the maximum threshold value of investors before the crisis occurs.

The estimation shows that the number of investors influenced by negative messages when the government publishes relevant policies, such as a monetary or fiscal policy to prevent a financial crisis, is lower than the number of investors influenced by negative messages when the government does not take action. Therefore, government involvement in financial uncertainty may contribute to reducing the possibility of a crisis occurring.

Wu et al. (2020) mention that since the emergence of the SARS-CoV-2 pathogenic virus in Wuhan, China, in 2019 and its spread to all countries, governments have been under great pressure to try to stop and control the outbreak with preparation and transparency as information sharing is crucial for health risk control. According to the author, this information should include outbreak reports, diagnoses, treatments, and prevention policies.

Wangping et al. (2020) state that with the spread of COVID-19 throughout the world and in the particular case of Italy, one of the countries that has suffered the most from the pandemic, the results have been devastating and mention that studying this case will help to establish health policies in each country. They recommend the establishment of stricter measures for the management of the disease as quickly as possible.

The paper by Boldog et al. (2020) suggests that the risk of a larger outbreak in countries depended on the evolution of the cumulative number of cases in China outside the closed areas, each country’s transport links with China, and the R0 contagion factor, as well as the number of imports or travel restrictions in countries with high contagion.

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Stylized facts

The outbreak of the COVID-19 coronavirus and its spread around the world has had many consequences for the global economic and financial system. Figure 1 shows the growth of contagions worldwide in contrast to the search for the term COVID-19 on the Internet, specifically in the Google search engine, where it can be seen that it is the official announcements about the pandemic and its possible consequences of economic and financial situations that have more immediate and decisive effects on the contagion of information in general and in particular for financial traders, even more than the actual growing number of sick people around the world, at least in the short term.

The arrival of the epidemic in the United States and Europe, as well as the declaration of a pandemic by the World Health Organization (WHO), the downward movement of the interest rate by the Federal Reserve System (FED) and the oil war, caused excessive growth of searches on the Internet for the word. On the other hand, the figures show a rapid increase in searches and a subsequent gradual fall where the actual growth in the number of infections is still ongoing; there is a minimal correlation between these two variables.

Figure 2 shows a high negative correlation (-0.83) between Google searches by the general population in the world for the word COVID-19 (in particular, it may be representative of traders) and economic and financial announcements, as well as pandemic announcements, with the drop in world markets represented by a typical global index such as the Dow Jones (DJI). In other words, the real disease pandemic can possibly be transmitted to a financial pandemic from traders' actions via instant communication on social networks or the Internet.

Figure 3 presents a scatter diagram showing the negative correlation adjusted by linear regression between the DJI index and Google searches. It is worth noting that there is an accumulation in two groups of data (the basic statistics of the regression can be consulted in the same figure).

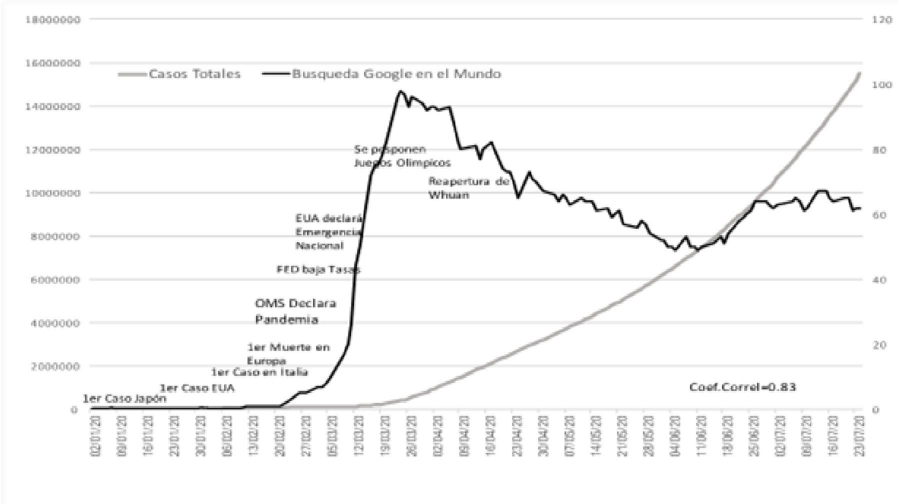

Figure 1. Growth of infections and effect on Google searches for COVID-19 in the world
Source: created by the author with data from https://ourworldindata.org/coronavirus-source-data and Google Trends

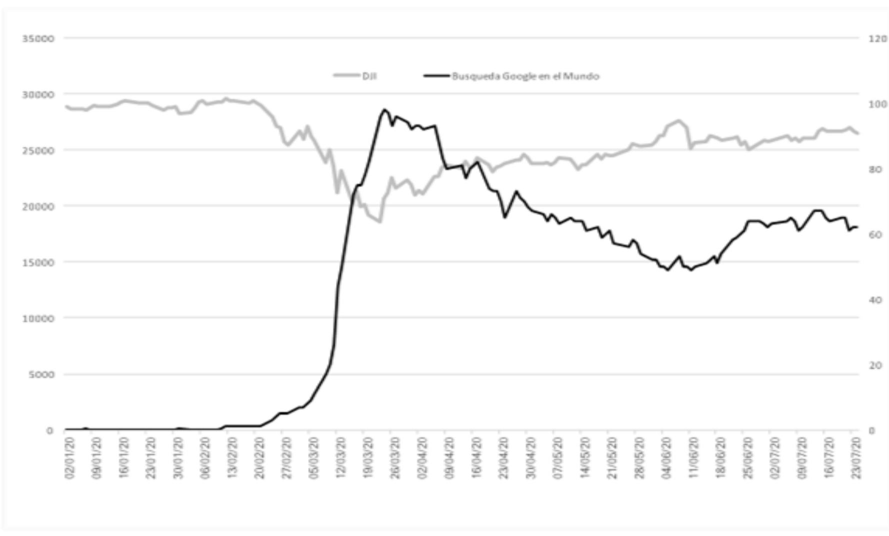

Figure 2. Google search for the word COVID-19 and the performance of the DJI Stock Index in the world
Source: created by the author with data from Yahoo Finance and Google Trends

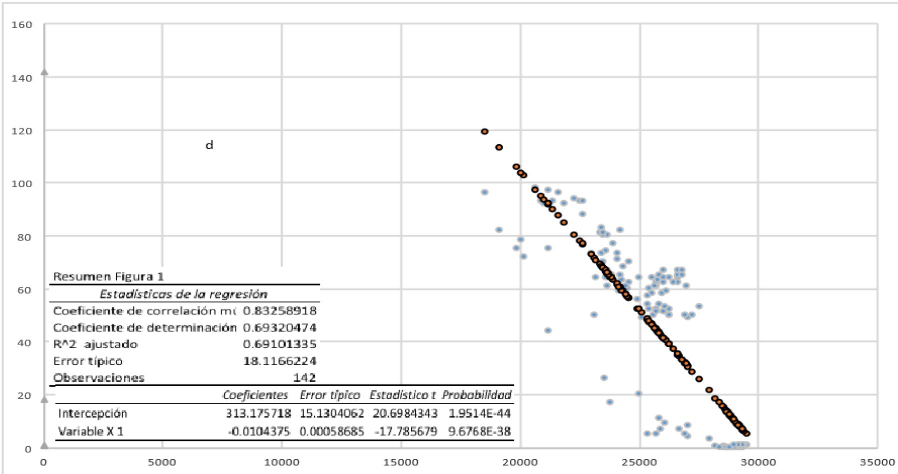

Figure 3. Dispersion effect on Google searches and DJI Stock Index in the world
Source: created by the author with data from Yahoo Finance and Google Trends

The same exercise can be repeated specifically for the United States (U.S.) in Figures 4, 5 and 6. Figure 4 shows the growth of COVID-19 infections in the USA against Google searches (COVID-19), also in the USA. Figures 5 and 6 show the same Google searches against the typical US DJI index and a scatter plot. The regression statistics are also presented (Figure 6), but surprisingly, the correlation is lower than the global one of the previous cases. Nevertheless, it is again observed how the effects of the contagion in the pandemic are transmitted to contagion in the markets through the search for information on the Internet.

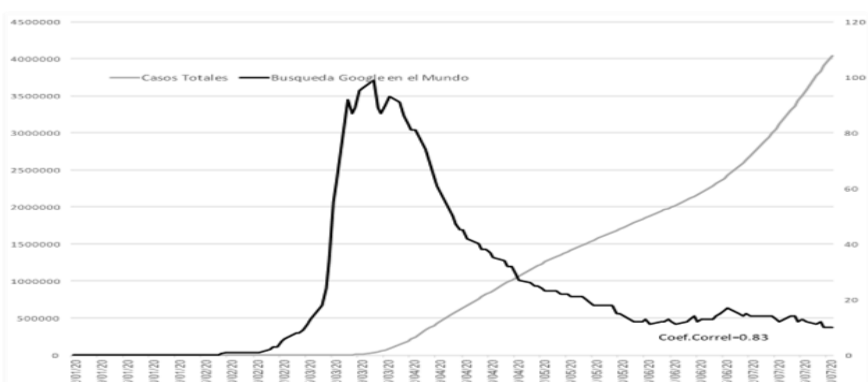

Figure 4. Contagion growth and effect on Google searches for COVID-19 in the United States
Source: created by the author with data from https://ourworldindata.org/coronavirus-source-data and Google Trends

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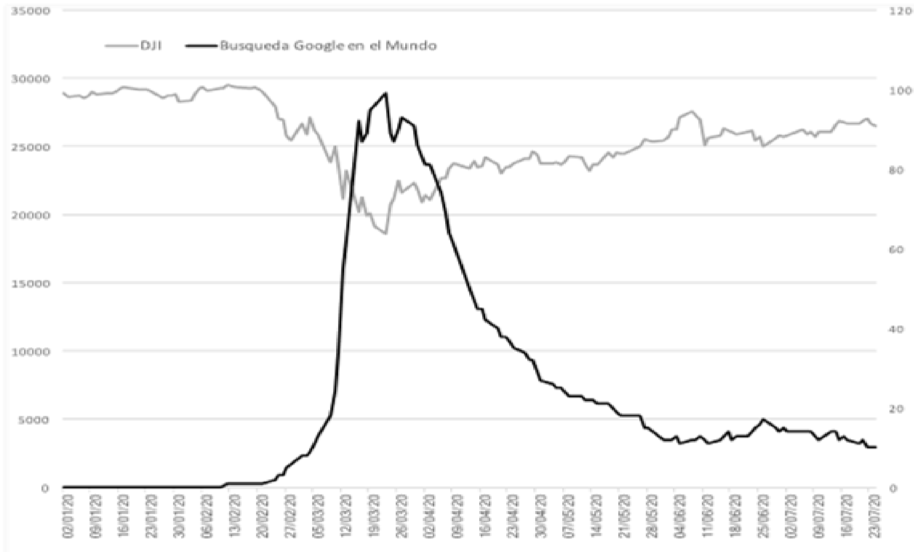

Figure 5. Google search for the word COVID-19 and the performance of the DJI Stock Index in the U.S.
Source: created by the author with data from Yahoo Finance and Google Trends

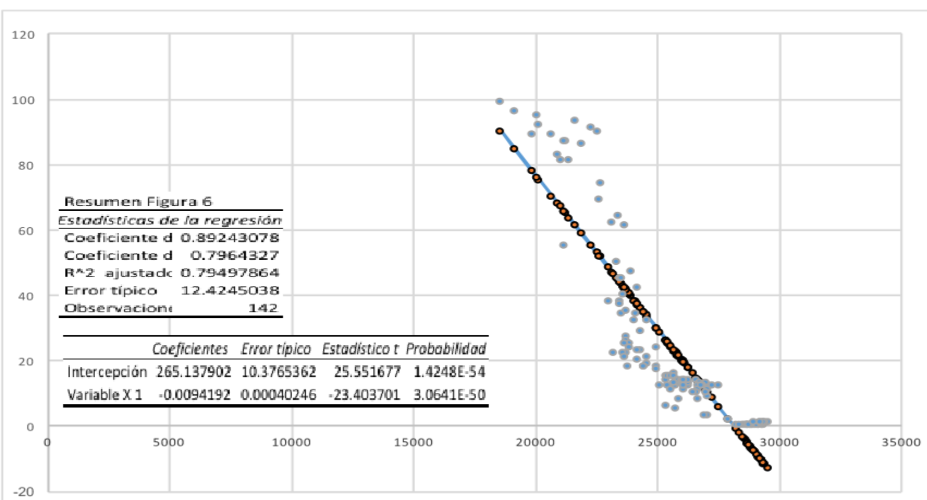

Figure 6. Dispersion Effect on Google Searches and DJI US Stock Indexes
Source: created by the author with data from Yahoo Finance and Google Trends

Finally, the same exercise is repeated for the case of Mexico in Figures 7, 8 and 9, and the contrast between searches for the word COVID-19 in Google and COVID-19 infections is observed. Likewise, in the searches against the representative index of Mexico (IPC), the correlation found is -0.91.

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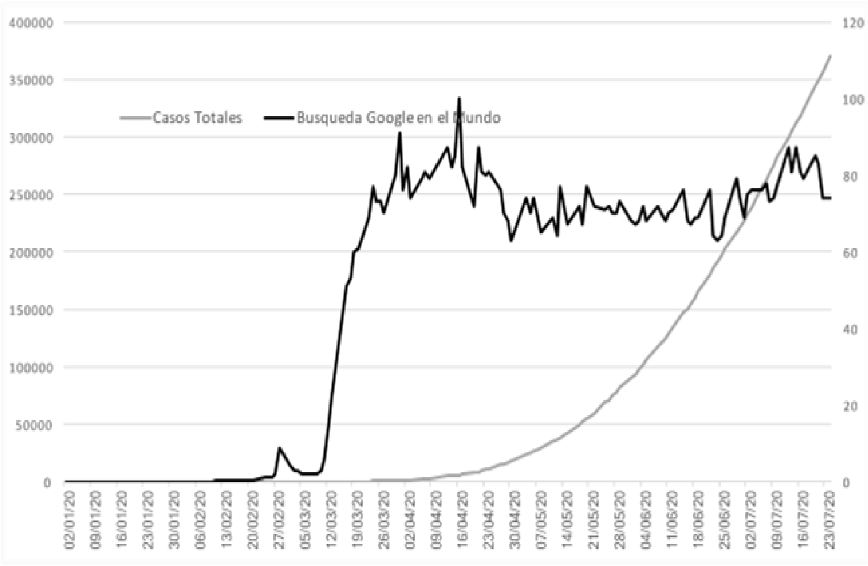

Figure 7. Contagion growth and effect on Google searches for COVID in Mexico
Source: created by the author with data from https://ourworldindata.org/coronavirus-source-data and Google Trends

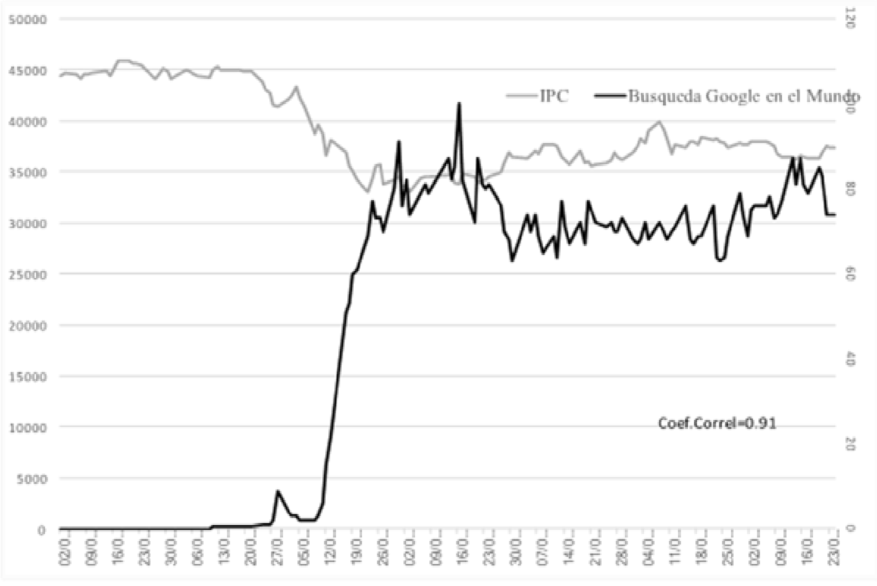

Figure 8. Google searches for the word COVID and the performance of the IPC Stock Index in Mexico
Source: created by the author with data from Yahoo Finance and Google Trends

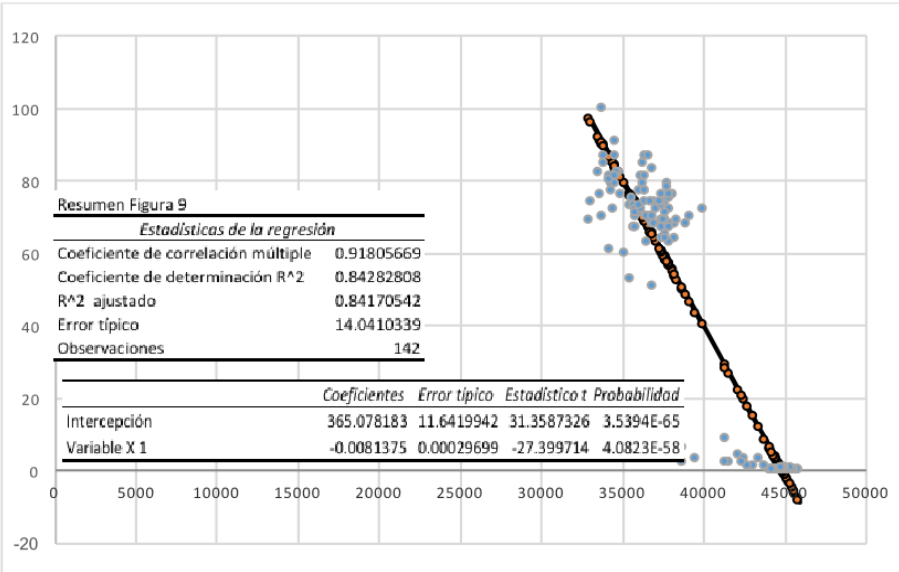

Figure 9. Dispersion Effect on Google Searches and CPI Stock Index in Mexico
Source: created by the author with data from Yahoo Finance and Google Trends

From the above figures, in particular Figures 3, 6 and 9, a negative correlation can be inferred in the world, USA and Mexico between searches for the word COVID-19 and the respective market indices.

Model

This section provides a review of the basic epidemiological models that will serve as the basis in the following section for the extension to a model to describe contagion from a disease epidemic to a financial environment contagion. This proposal will consider the stylized facts of the previous section and construct a deterministic and Monte Carlo simulation model for the transmission of the pandemic in financial markets.

An infectious disease is considered as such when it is caused by an agent, virus, bacterium, toxin, etcetera, and can pass from one host organism to another through a mode of transmission of direct contact, such as air, water, food, or from mother to offspring or others.

According to Li (2018), mathematical models of contagious diseases help in understanding the mechanisms of transmission and spread of infectious diseases and can suggest better pandemic control, prevention measures, and an estimation of the severity and potential scale of the epidemic, keeping in mind that the models approximate reality. In general, there are three different approaches to the mathematical modeling of infectious diseases:

1) Statistical models are essential for data utilization and are widely used by epidemiologists and public health researchers.
2) Deterministic models typically use differential or difference equations and assume that changes in populations are continuous over time and mainly describe the interrelationships between rates of change.
3) Stochastic models, where diseases are treated as stochastic processes and the models describe the interrelationships of their probability distributions among populations.

Among the deterministic models is the compartmental approach, where the total host population is divided into exclusive groups according to the evolution of the disease, usually the susceptible (S), infected (I), and recovered (R); these models are generally called SIR.

Li (2018) also states that the simplest deterministic model (SIR) is that of Kermack and McKendrick (1991), which considers the following assumptions of the host or host population:

1) Transmission occurs horizontally through direct contact between hosts.
2) The combination of individuals is homogeneous, and the number of contacts between hosts in different compartments depends only on the number of hosts in that compartment (obeying what is known as the law of mass action[1]). The number of infections per unit time or incidence rate can be expressed by $\lambda I(t)S(t)$, where λ is known as the transmission coefficient.
3) The compartment transfer rate is proportional to the size of the compartment population. For example, the transfer rate from infected (I) to recovered (R) can be written as $\gamma I(t)$. Infected individuals become infected without a latency period.
4) There is no loss of immunity, and the possibility of reinfection implies that the transfer rate of recovered (R) to healthy population (S) is zero.
5) There is zero influx of new susceptible individuals, and they are not removed from any compartment.
6) At all times, the total host population is constant.

Using the hypotheses and variables of the previous terms, a more basic SIR mathematical model can be proposed with the following system of differential equations:

[1]This law is based on Waage Gullberg's (91864) Law of Mass Action of Chemical Kinetic Energy and states that the rate of chemical reaction is proportional to the reactant density reaction A + B ⟺ AB s rate = k[A] [B]

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$$\frac{dS}{dt} = -\lambda IS \tag{1a}$$

$$\frac{dI}{dt} = \lambda IS - \gamma I \tag{1b}$$

$$\frac{dR}{dt} = \gamma I \tag{1 c}$$

with initial conditions S(0) = S0 > 0, I(0) = I0, R(0) = 0

where the functions are as follows: S(t) is the host population, I(t) is the infected population and R(t) is the recovered host population, and λ and γ are models of the parameter y where the initial time is at time zero. The interpretation of the above system is very straightforward with three compartments. The increase of the infected will be according to the size of the infected and susceptible group via the transmission coefficient eq. (1 b) Moreover, the same proportion would decrease the proportion of possible susceptible eq. (1 a). On the other hand, the increase in the recovered will be according to the number of infected with the proportion of the transfer rate eq. (1c) Furthermore, the same proportion will decrease the possible increase in infected ec. (2).

Where the total population does not change and remains constant, N(t)= S(t)+I(t)+R(t) with N(t)= N0=S0+I0+R0 and dN/dt= 0. If a review of the behavior of the growth or non-growth of a pandemic is performed, the following scenarios must be considered:

If dS/dt ≤ 0 , S(t) is always decreasing, then a pandemic will not occur, in particular S(t) ≤ S0.

If S0 <Y/λ then dI/dt |t=0 < 0, therefore I(t) decreases strictly and the pandemic does not occur.

If S0 > Y/λ , then S(t) > Y/λ therefore I' (t) > 0, grows strictly, and the pandemic occurs.

R0 is the reproduction number or reproductive rate and is an important parameter in epidemic modeling. It measures the average number of secondary infections caused by an individual infection in a susceptible population during the mean of the infection period.

The most basic model above can be complemented by considering the entry and exit of potential hosts through migrations, births, and deaths, including periods of latency and incubation, as well as the introduction of periods of lockdown, among others.

Proposed model

From the stylized facts in section 3 it can be seen that the COVID-19 pandemic outbreak that originated in Wuhan in China and spread worldwide as a contagious disease can be modeled with Kermack and McKendrick (1991) to determine the behavior of the host population, infected, and recovered from the three Equations (1 a), (1 b), and (1 c). On the other hand, the world's financial markets in certain periods present contagion behavior both for the growth of prices (bubbles) and their subsequent fall, giving rise to financial crises in any of the different types of financial assets (e.g., energy, stocks, derivatives, etcetera).

Throughout the history of the world economy, bubbles and crises have existed. They have arisen for different reasons but are often triggered by purely financial events such as currency overvaluations, fraud in operations, and shortages of goods. Still, they can also be caused by pandemics.

Contagious diseases are usually transmitted by microscopic organisms that overwhelm the immune system as they are new or have undergone mutations. There is no such virus in the financial markets, but there is undoubtedly a contagion criterion for the massive buying or selling of certain financial assets. Despite the proposed models, this form of transmission has not been easy to describe (see sections 2 and 3). A good approximation is that of Fan and Huang (2017), where an equivalent of the Kermack and McKendrick model (1991) is considered for financial markets, where individuals susceptible to being influenced by negative news are S(t), those infected by bad news are I(t), and those recovered from that bad news are R(t). The system of equations proposed by the authors for this financial contagion is similar to Equations (1 a), (1 b), and (1 c).

This paper proposes the notion of contagion through communication, which nowadays, with the globalization of communications, occurs almost instantaneously among groups of traders worldwide through social media and the Internet. In this case, it is proposed that transmission occurred from the disease pandemic to the financial pandemic, with the Internet as the propagation organism. The degree of relation between the growth in contagion, internet searches, and market movements in Mexico and worldwide is very high, as mentioned in the stylized facts in section 3.

A generalization of the SIR model in Li (2018) and the model of Kermack and McKendrick (1991) and Fan and Huang (2017) is proposed, where a system of equations is established, in which the first three equations correspond to the description of the COVID-19 pandemic (equivalent to Equations 1 a), 1 b) and 1c)) and the following 3 are a system of equations for the description of the financial contagion in traders seeking information on social networks and especially on the Internet. The effect of susceptible, infected, and recovered individuals in the financial markets has the same logic as the first set of equations, except that in addition to the dynamics of financial contagion itself, the effect of the number of those infected by the disease pandemic is incorporated, which is an essential variable that connects and feeds back into the system of the last three equations.

Different combinations are suggested for modeling the behavior of the traders, considering how the characteristics of the pandemic influence them. It should be noted that transmission in the financial markets is instantaneous, while transmission of the disease is person-to-person through proximity and physical contact. The model is presented below with very similar characteristics to those of Kermack and McKendrick (1991), in addition to the equations for the description of contagion in financial markets.

$$\frac{dS_1}{dt} = -\lambda_1 I_1 S_1 \tag{2 a}$$

$$\frac{dI_1}{dt} = \lambda_1 I_1 S_1 - \gamma_1 I_1 \tag{2 b}$$

$$\frac{dR_1}{dt} = \gamma_1 I_1 \tag{2 c}$$

$$\frac{dS_2}{dt} = -\lambda_2 (I_2 + I_1) S_2 \tag{2 d}$$

$$\frac{dI_2}{dt} = \lambda_2 (I_2 + I_1) S_2 - \gamma_2 (I_2 - I_1) \tag{2 e}$$

$$\frac{dR_2}{dt} = \gamma_2 (I_2 + I_1) \tag{2 f}$$

with initial conditions S1(0) = S01 > 0, I1(0) = I01,R1(0) = 0 S2(0) = S02 > 0, I2(0) = I02,R2(0) = 0. It is important to remember that in the financial epidemic (S2, I2 and R2), in addition to the financial variables themselves, the effect of those infected by the pandemic is included.

An additional effect can be added to the previous model by creating a new compartment or group within the system of equations of financial contagion (variables denoted with subscripts 2), with remediated investors G(t) or traders who no longer believe in the negative messages that are appearing because governments have already made public their fiscal and monetary policies for managing the crisis and in this way try to regain the confidence of the markets. It has to do with the size of the infected population and the size of the same group of remediated agents. Then a new compartment of the financial contagion group G is created that will affect those affected and recovered from the financial contagion with the same logic, Equations (3 f) and (3g).

$$\frac{dS_1}{dt} = -\lambda_1 I_1 S_1 \tag{3 a}$$

$$\frac{dI_1}{dt} = \lambda_1 I_1 S_1 - \gamma_1 I_1 \tag{3 b}$$

$$\frac{dR_1}{dt} = \gamma_1 I_1 \tag{3 c}$$

$$\frac{dS_2}{dt} = -\lambda_2 (I_2 + I_1) S_2 \tag{3 d}$$

$$\frac{dI_2}{dt} = \lambda_2 (I_2 + I_1) S_2 - (\alpha_2 + \gamma_2)(I_2 - I_1) \tag{3 e}$$

$$\frac{dR_2}{ddt} = \alpha_2 (I_2 - I_1) + \eta_2 G_2 \tag{3 f}$$

$$\frac{dG_2}{dt} = \gamma_2 (I_2 - I_1) - \eta_2 G_2 \tag{3 g}$$

An additional variant of the system that mainly links health contagions to the previous system of equations is also considered, incorporating a lockdown compartment, which is a process where the infected group is isolated to avoid infecting the others and thus limit or stop the transmission process. Therefore, a new variable is created that will only affect the health contagion group, especially Equation (4 c), where the infected, besides going into recovery, could go into lockdown Q and from the lockdown group return to the recovery group. Hence, a new Equation (4 d) is created and its effect on the recovered will not affect the financial contagion system. The following is the new system of differential equations with the latest modification:

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$$\frac{dS_1}{dt} = -\lambda_1 I_1 S_1 \tag{4 a}$$

$$\frac{dI_1}{dt} = \lambda_1 I_1 S_1 - \gamma_1 I_1 \tag{4 b}$$

$$\frac{dR_1}{dt} = \gamma_1 I_1 + \phi_1 Q_1 \tag{4 c}$$

$$\frac{dQ_1}{dt} = \delta_1 I_1 - \phi_1 Q_1 \tag{4 d}$$

$$\frac{dS_2}{dt} = -\lambda_2 (I_2 + I_1) S_2 \tag{4 e}$$

$$\frac{dI_2}{dt} = \lambda_2 (I_2 + I_1) S_2 - (\alpha_2 + \gamma_2)(I_2 - I_1) \tag{4 f}$$

$$\frac{dR_2}{dt} = \alpha_2 (I_2 - I_1) + \eta_2 G_2 \tag{4 g}$$

$$\frac{dG_2}{dt} = \gamma_2 (I_2 - I_1) - \eta_2 G_2 \tag{4 h}$$

The above models are deterministic models based on systems of differential equations, each including different groups or compartments. An alternative approach is then proposed. This alternative involves modeling through simulations, specifically Monte Carlo simulations.

The Montecarlo method solves mathematical problems applied to different sciences and engineering using the simulation of random variables. The method is very powerful because it allows the simulation of any process whose progress depends on random factors and other mathematical problems that have no relation with random questions but can create a dependence on the artificial probability that allows problems to be solved.

Although the theoretical basis of the method had been well-known for a long time, it was not until the development of computers that it began to gain momentum. The birth date of the Monte Carlo

Method is considered to be 1949. The creation of this method is usually attributed to the American mathematicians Von Neumann (1949) and Metropolis and Ulam (1949). The method has two important characteristics:

The first is that its algorithm has a very simple structure. As a rule, an algorithm for performing the randomized test is first worked out. This test is repeated N times so that each experiment is independent of the others, and the mean of the results of the experiments is taken. The second characteristic is that the error is, as a rule, proportional to the magnitude √(D/N) where D is a constant and N is the number of tests. In other words, the error is decreased 10 times by 100 times. The problem is solved by looking for different ways to adjust D.

There may be different ways of mathematically modeling the behavior of the growth of infected in a pandemic. In an alternative approach in this work, it will now be proposed that it behaves similarly to the growth of financial prices. In other words, the increase in prices at time t (St) will be composed of a growth trend μSt and a stochastic component as a geometric Brownian motion $\sigma StdWt$ in a risk-neutral world, according to:

$$dSt = \mu Stdt + \sigma StdWt \tag{5}$$

Where the first term is the trend and the second is the stochastic component where Wt is a Brownian motion (W (t)) t > 0 (also known as the Wiener process) in a probability space, with a filtration with the following properties:

i) Wt (0) = 0
ii) If 0< t1 < t2 < … < tn, then W (t2) - W (t1), W (t3) - W (t2), …, W (tn) - W (t n-1) are independent random variables
iii) If s < t, then W (t) - W (s) is distributed as N (0; t- s)

An alternative expression, after a logarithmic transformation of the equation, is as follows:

$$S(t+\Delta t) = S(t)\,exp\left((r-\frac{1}{2}\sigma^2)\Delta t + \sigma\Delta W\right) \tag{6}$$

If the above expression is used for an equivalent one for the case of the number of contagions, it would generate the number of infections recurrently over time (see Fan and Huang (2017)) but with a growth trend ($\lambda 1$ - $\gamma 1$)1t and a stochastic term $\sigma 1I1tdWt$ with the parameters involved in the infections:

$$dIt = (\lambda 1 - \gamma 1)1tdt + \sigma 1I1tdWt \tag{7}$$

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or equivalently, for a description of growth in health pandemic infections:

$$I_1(t+\Delta t) = I_1(t)\,exp\left(((\lambda_1-\gamma_1)-\frac{1}{2}\sigma_1^2)\Delta t+\sigma_1\Delta W\right) \tag{8}$$

and for the case of an increase in the pandemic in financial through the transmission of information it would result in something equivalent to the following[2] :

$$I_2(t+\Delta t) = I_2(t)\,exp\left(((\lambda_2-\gamma_2)-\frac{1}{2}\sigma_2^2)\Delta t+\sigma_2\Delta W\right) \tag{9}$$

Nevertheless, after carrying out a few exercises, the proposal as to where the growth of the health pandemic influences the growth of contagions in the financial environment would be as follows:

$$I_2(t+\Delta t) = \sqrt{I_2(t)I_1(t)})\,exp\left(((\lambda_2-\gamma_2)-\frac{1}{2}\sigma_{12}^2)\Delta t+\sigma_{12}\Delta W\right) \tag{10}$$

which is a different view of the pandemic's effect on financial market contagion. After reviewing the deterministic and simulation models, the following section shows the results obtained for both models.

Results

Once the cases of the deterministic differential equation models of the previous section have been presented, the next step consists in solving each of the proposed systems that describe the dynamics of the evolution of the COVID-19 epidemic and their transmission to the financial epidemic. These systems are not easy to solve analytically, so that numerical approximations will be solved with software and programming developed in Python. It should be clarified that the parameters of the models vary over time and from region to region, so some standard parameters are shown as a reference and others are calibrated to emphasize the effects. They do not represent any particular country or period, especially the financial parameters, but some of them were taken with some references such as those in Martin (2020).

From the exercises presented, an example of the SIR model is taken as a base and for the parameters $\lambda 1 = 0.000167{,}1 = 0.00167$, the following is the typical graph, in which the increasing behavior

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[2] Sub-indexes 1, 2 will refer to the disease and financial pandemics respectively

of the infected (It), the recovered (Rt) and the host population (St) can be seen, as well as how it decreases as the infected grow in Equations (1 a), (1 b), (1 c), see Figure 10.

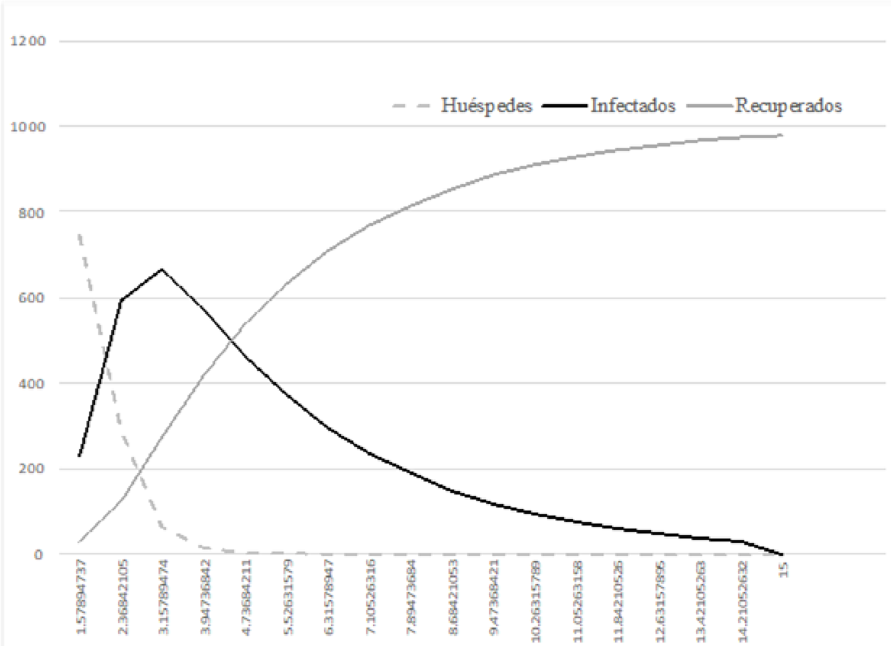

Figure 10. Original SIR model
Source: created by the author with Equations (1 a), (1 b), and (1 c)

The disease epidemic is transferred to the financial epidemic with a system of six differential Equations (2a), (2b), (2c), (2d), (2e), and (2f) describing the evolution of the host population (S1t), infected (I1t), recovered R1t, and host traders (S2t), as well as those contaminated with bad news (I2t) and the recovered R2t. As mentioned above, after testing various parameters, the typical ones that fall within the intervals and emphasize the effects can be seen in Figure 11. $\lambda1 = 0.000167$, $\gamma1 = 0.00167$,$\lambda2 = 0.00167$, $\gamma2 = 0.0167$. Moreover, it can be observed that the behavior is similar to that of the stylized facts in Figures 1, 4, and 7.

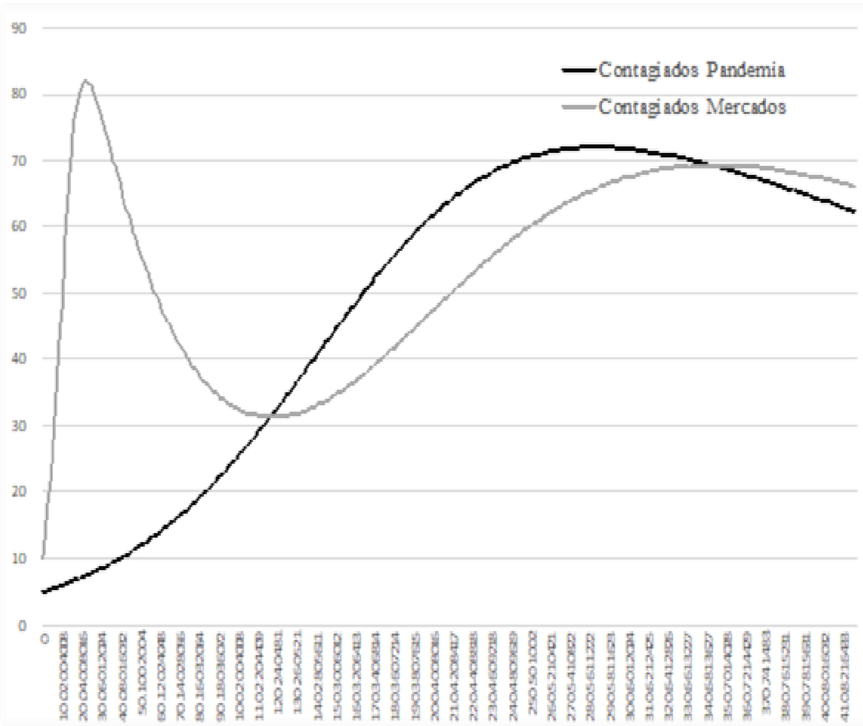

Figure 11. Model 1 Contagion pandemic to markets
Source: created by the author with Equations (2 a), (2 b), (2 c), (2 d), (2 e), (2 f)

In the following systems of Equations (3 a),(3 b), (3 c), (3 d), (3 e), (3 f), and (3 g), a compartmental group of traders who are no longer affected by bad news (Figure 12) is added to the case of the financial pandemic with the following parameters: $\lambda 1 = 0.0029$, $\gamma 1 = 0.29$, $\lambda 2 = 0.5$, $\gamma 2 = 0.29$, $\alpha 2 = 0.2$, $\eta 2 = 0.1$, $\lambda 1 = 0.000167$, $\gamma 1 = 0.00167$, $\lambda 2 = 0.00167$, $\gamma 2 = 0.0167$, $\alpha 2 = 0.01$, $\eta 2 = 0.1, 0.01, 0.001$

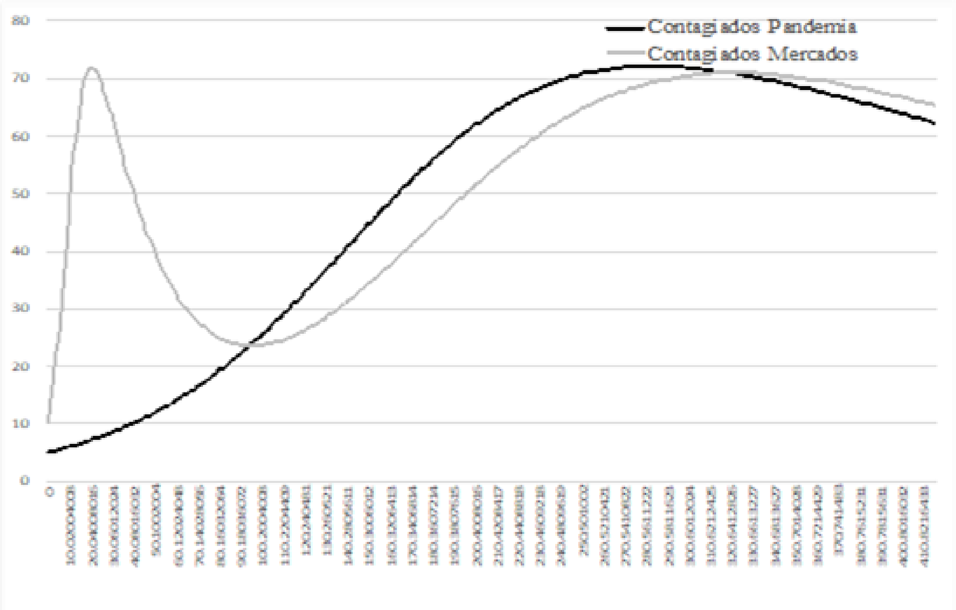

Figure 12. Model 2 Contagion pandemic to markets
Source: created by the author with Equations (3a), (3b), (3c), (3d), (3e), (3f), (3g)

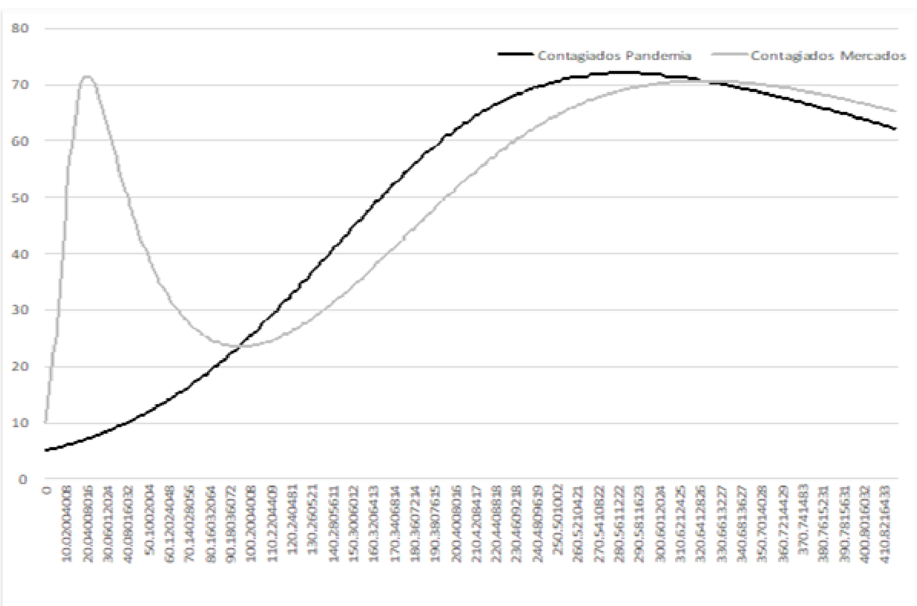

Figure 13. Model 3 Contagion pandemic to markets
Source: created by the author with Equations (4a), (4b), (4c), (4d), (4e), (4f), (4g), (4h)

In the last group, a new compartmental group is added where the individuals that are sent into lockdown with Equations (4a), (4b), (4c), (4d), (4e), (4f), (4g), (4h) are located. The parameters are as follows (Figure 13): $\lambda 1 = 0.000167, \gamma 1 = 0.00167$, $\phi 1 = 0.001, \delta 1 = 0.01$, $\lambda 2 = 0.00167, \alpha 2 = 0.01, \gamma 2 = 0.0167$, $\eta 2 = 0.001, \alpha 2 = 0.2, \eta 2 = 0.001$.

As mentioned above, the behaviors mentioned earlier can be applied to different initial conditions and parameters where they are of special interest. It is worth mentioning that the effects of skeptical traders and the lockdown change the results very little.

On the other hand, using the second modeling proposal with the Monte Carlo method, the scenario of the growth of infected people in the pandemic combined with the scenario of contagion in the financial market is generated, and the parameters can be adjusted to a certain period or country. After programming again in Python, scenarios are generated with the parameters shown in Table 1 and Figures 14 and 15.

Table 1
Parameters of disease contagion to financial contagion using the Monte Carlo method

	Pandemic Contagion	Financial Contagion
Initial Infected	10	20
Infections Growth Mean	5	5
D.E. of infections	0.10	0.70
Period	1 year	1 year
No. Simulations	10	10
Limit start of financial contagion	100	

Source: created by the author

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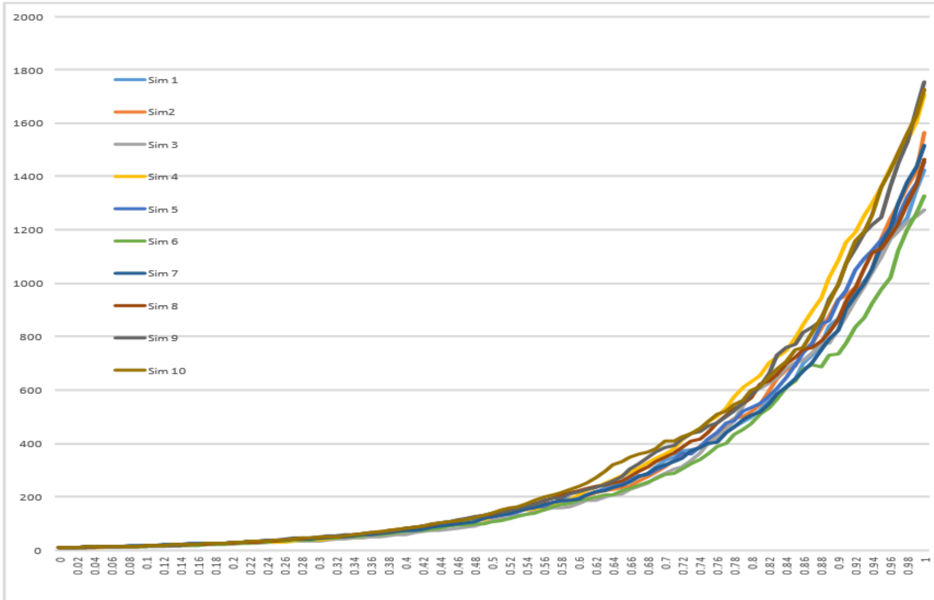

Figure 14. Monte Carlo simulation of pandemic growth
Source: created by the author

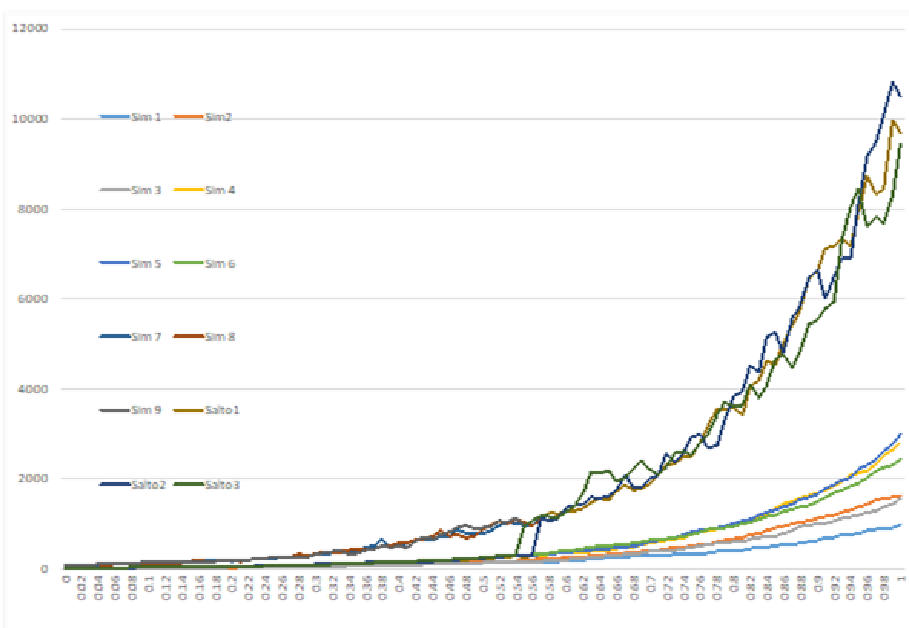

Figure 15. Disease pandemic contagion in the market with Monte Carlo simulation
Source: created by the author

As shown in Figures 14 and 15, there is rapid growth in the financial contagion supported by the media. Once the upper limit imposed by the disease contagion is reached, the financial pandemic begins to expand rapidly.

Conclusions

Despite the unfortunate situation that occurred worldwide in 2020 as a result of the COVID-19 outbreak in terms of health (especially in terms of infections and deaths) and its economic consequences, it is possible to propose models and analyze the behavior of populations in order to understand and explain some of the situations that occurred in the world. As stated in this paper, a proposal is made to explain the relations between disease pandemics and financial crises, fulfilling the initial objective and hypotheses of modeling a financial contagion from the initial effect of a pandemic using a system of differential equations and a Monte Carlo simulation.

Before financial crises, there is an exogenous trigger that initiates the moment of contagion among market players and drives participants to move in one direction in buying and selling financial assets. In the case of 2020, COVID-19, the infectious disease is transmitted from person to person through a virus and has thereby spread around the world.

The contagion of an infectious disease can jump to the contagion of financial markets not by a biological virus but rather by contagion in electronic media such as the Internet and social networks. Examples include internet searches (Google) for the word COVID-19, which alert market agents to the increasing number of infections and deaths in the pandemic, as well as the economic consequences that will ensue from it.

From the proposal of deterministic differential equation models, the modeling of a health pandemic is extended to financial contagion, taking the SIR model as an initial basis and expanding it to include more cases. The graphs show that the behavior of the pandemic contagion and the transmission by electronic means and subsequent contagion to the markets is very similar to that shown in the stylized facts (Figure 11). The peak of financial contagion is reached much faster than disease contagion. In other words, the transmission of contagion in crises is much quicker.

Additionally, as an extension to the model, a compartment or group is incorporated in the financial contagion of traders that no longer respond to contagion due to the lack of certainty caused by government policy announcements that will no longer have effects. Finally, a new compartment or variable is added to the disease contagion of the lockdown situation. The observed outcome of both effects is very similar to the behavior of the first systems, especially in the short term, and the maximum financial contagion is reached first. The model can be extended to other variables, parameters, and specific periods in future research.

The behavior of a geometric Brownian motion for the variable of disease infections and markets in the short run is also very similar to that observed. Despite the small number of simulations, it is possible to follow the jump in the number of financial infections when a limit of disease contagion is reached, in other words, the contagion effect. The parameters can also be changed for different countries or periods.

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