Measuring the operational risk for health service providers in the Covid-19 era

Medición del riesgo operacional para entidades promotoras de salud en tiempos de la Covid-19

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Abstract

The objective of the research is to show the importance of using advanced measurement methods (AMA), for the quantification of the operational Value at Risk (OpVaR), in order to strengthen the risk management systems (SAR) in the sector of the health even in times of pandemic such as Covid-19. Based on a case analysis with information from 2019 and 2020, the implementation of two advanced measurement methods is presented; Monte Carlo simulation (SMC) and Böcker and Klüppelberg analytical approach. The results with SMC show that the monthly Operational Value at Risk for a confidence level of 99.9%, is given by COP 3861.39 million for the year 2019 and by COP 5052.80 million for the year 2020, showing an increase of 30.85%. Additionally, the application of the Böcker and Klüppelberg approximation is illustrated and its sensitivity to changes in the levels of trust and behavior of the severities in the tail of the distribution is evidenced. The limitations are that the proposed models for frequency and severity are classical probability distributions. The originality is that the analysis in period of the Covid-19 is considered. It is concluded that the proposed quantitative approach can be replicated as an internal AMA method in any health entity worldwide to strengthen its SARs and that increases in OpVaR could also be expected for the next few years given that many losses materialize with lags of years.

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Resumen

El objetivo de la investigación es mostrar la importancia de utilizar métodos de medición avanzada (AMA), para la cuantificación del Valor en Riesgo operacional (OpVaR), con el fin de robustecer los sistemas de administración del riesgo (SAR) en el sector de la salud aún en tiempos de pandemia como la Covid-19. A partir de un análisis de caso con información del 2019 y 2020, se presenta la implementación de dos métodos de medición avanzada; simulación Monte Carlo (SMC) y aproximación analítica de Böcker y Klüppelberg. Los resultados con SMC muestran que el Valor en Riesgo operacional mensual para un nivel de confianza del 99.9%, está dado por COP 3861.39 millones para el año 2019 y por COP 5052.80 millones para el año 2020, mostrando un incremento del 30.85%. Adicionalmente, se ilustra la aplicación de la aproximación de Böcker y Klüppelberg y se evidencia su sensibilidad ante cambios en los niveles de confianza y comportamientos de las severidades en la cola de la distribución. Las limitaciones son que los modelos propuestos para frecuencia y severidad son distribuciones de probabilidad clásica, la originalidad es que se considera el análisis en periodo de la Covid-19. Se concluye que el enfoque cuantitativo propuesto puede ser replicado como método interno AMA en cualquier entidad de salud a nivel mundial para fortalecer sus SAR y que además se podrían esperar incrementos del OpVaR para los próximos años dado que muchas pérdidas se materializan con rezagos de años.

Código JEL: C11, C15, H51
Palabras clave: Riesgo operacional; sector salud; simulación Monte Carlo; aproximación de Böcker y Klüppelberg

Introduction

The materialization of operational risks in the health sector is an issue of global interest and concern that has grown with the Covid-19 pandemic. According to the World Health Organization (WHO, 2019), patient safety is a serious public health problem worldwide; it is estimated that, on average, globally, 1 in 300 patients is affected by adverse events directly caused by medical care and 14 out of every 100 hospitalized people contract an infection. According to Singh et al. (2014), in the United States, medical diagnostic errors affect at least 1 in 20 adults. In addition to the above, the Covid-19 pandemic has brought new challenges to be taken into account: firstly, it has triggered a transformation toward virtual health care (Wosik et al., 2020); additionally, it has provided evidence that the work system in the health sector, particularly in primary care, is chaotic and stressful, which leads to burnout of medical staff and may increase the possibility of negative impacts on patient care (Olson et al., 2021).

The situation caused by the Covid-19 pandemic has put the health systems of all countries worldwide under maximum levels of stress, increasing the possibility of the occurrence of adverse events, defined as normally preventable consequences for the patient's health, that are more likely to be caused by medical care than by the underlying disease, and that may result in death, disability, or prolonged hospitalization. Examples of preventable adverse events that may occur to patients in a Health Service

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Provider (HSP) include errors in medical diagnosis, falls out of bed, failures in medical procedures, errors with radiation equipment, and inadequate supply of drugs, among others.

The materialization of risks in the health sector may have consequences such as compromising patients' lives and financial losses due to subsequent lawsuits. For example, according to WHO (2017), the global cost associated with medication errors is US$ 42 billion annually, corresponding to approximately 1% of global healthcare expenditure. In addition, some research places deaths caused by medical errors as one of the leading causes of death in the United States (Anderson & Abrahamson, 2017). As explained by Vishnu et al. (2019), risk management in the health sector is of great relevance and is also a crucial area of research from a humanitarian perspective. All of the above demonstrates the importance of creating and consolidating Risk Management Systems (RMS) in health service providers in all countries, especially when many do not even have historical records corresponding to the materialization of risks.

At the international level, and after the creation of the International Convergence of Capital Measurement and Capital Standards, or Basel Accords, issued by the Bank for International Settlements (BIS), the quantification of risk has become essential for corporations. The regulatory agencies of each country have been implementing regulations incorporating the quantification of risks (credit, market, and operational, among others) to calculate the solvency relation and estimate the capital requirements necessary in each case. Similarly, in the case of the health sector, the regulations and Ministries of Health of each country regulate the solvency margin that maintains the financial stability of the HSPs, corresponding to companies that provide medical services through clinics and hospitals, whether public, private, or mixed.

In recent years, there has been increased attention from decision-makers and researchers to the study of data quality for healthcare, use of tools and concepts such as corporate governance, internal auditing, and risk management in healthcare presentation (Biancone, 2019). Additionally, as stated by Capasso et al. (2019), risk management in the HSPs must identify, prevent, and manage risk to improve the quality of the health service; it is also necessary to evolve toward a culture of transparency in the analysis of errors, in order to identify critical points and be able to take corrective measures. To this end, they indicate four phases: identification of the risk profile of each area examined; establishment and application of prevention measures; activation of a control system; and proposal of progressive improvement for prevention.

Risk is the possibility of economic loss due to an adverse event. Concerning implementing a Risk Management System (RMS) for the health sector, the areas of action have traditionally been identification, evaluation and measurement, selection of management methods (avoidance, prevention,

retention, or transfer), implementation, and feedback. The HSP must ensure the application of the RMSs by guaranteeing the implementation of the cycle of planning, checking, and acting in the face of risk.

Regarding risk management methodologies, the regulations initially allow qualitative methods but require the application of technically valid quantitative methods in the final implementation phase to assess the identified risks. For the quantification of risks, it is essential to use adequate statistical, actuarial, and financial methods, and it is critical to have historical series of the HSP in order to be able to carry out the modeling. Following the RMS risk assessment, the entities must normally send technical reports to the governmental bodies monitoring and controlling the health system.

Operational risk is the potential loss of an entity due to failures or deficiencies in internal systems, processes, people, or some external factors. Unlike other sectors, such as the financial sector, which has standardized practices for quantifying risk, in the health sector there is a lack of conceptual standardization, a reactive culture, a lack of unified methodology, and a reluctance to make public the frequency and severity of losses generated by the historical materialization of operational risks. The characteristics mentioned above are commonly found in various health systems worldwide. Additionally, the literature concerning quantifying operational risk in the healthcare sector using advanced measurement methods is highly limited.

For regulatory bodies to approve operational risk management methods in health sector entities, they must be quantified using appropriate quantitative methods. As stated by Reina et al. (2010), implementing an RMS in the health sector requires a comprehensive approach that involves everything from business processes to the measurement of the impact on the health levels of members.

In turn, an Operational Risk Management System (ORMS) must be composed of a certain number of elements (policies, procedures, documentation, organizational structure, registration of operational risk events, control bodies, technological platform, information dissemination, and training) through which to achieve effective operational risk management. Particularly in HSPs, operational risk can generate three types of losses: losses in the health outcomes of the target population, in the expected operating results, including the satisfaction of members, and in the financial results of the entity and its controlling, related, and affiliated entities.

The article is arranged as follows. A brief literature review follows this introduction. Next, the Monte Carlo simulation methodology (SMC) and the so-called Böcker and Klüppelberg analytical approach for quantifying operational risk are described. Subsequently, a case study of an HSP is presented. The proposed methodology can be replicated and adapted internationally to structure an ORMS in any HSP. Finally, the results are presented, and some conclusions are drawn regarding modeling operational risk in the health sector.

Literature review

Key to the modeling of operational risk is the analysis of the frequency and severity distributions that give rise to the subsequent modeling of aggregate losses. As for loss distribution, Jantsch et al. (2019) analyzed operational risk control for 100 financial institutions in eleven Brazilian states. The results show potential risk factors: inadequate practices with clients, products and services, external fraud, poor workplace safety, and staff issues. On the other hand, Ramírez et al. (2021) consider an aggregate loss model with dependent losses, where from a real operational risk database and through Markov processes, the aggregate loss model is adjusted by maximum likelihood and severity through the lognormal Pareto distribution with two heavy tails. The results show that higher capital charges will be generated due to the dependence and overdispersion in the time distribution between losses.

Likewise, there are advanced models for measuring losses (AMA). For example, Rodríguez et al. (2009) use this type of model with a loss distribution approach (LDA), performing a stress-testing analysis on the operational risk capital of credit institutions, with the main outcome being that the loss severity distribution is greater than that of frequencies.

Based on the Basel Accords, Di Pietro et al. (2012) evaluate the operational risk quantification of frequency distribution and severity for a Spanish savings bank company. Subsequently, Macías et al. (2018) worked with the LDA model suggested by Basel. This model is applied to three types of operational risk events in a line of business of a Colombian financial institution. Through the characteristics of the expected and unexpected loss distributions, the Operational Value at Risk (OpVaR) is determined using two methods of comparison, with the main outcome being a threshold for the organization to maintain its required economic capital to cover future exposures to operational risk.

Regarding Bayesian network models, Martinez and Venegas (2013) identify and quantify the different operational risk factors of the settlement process in the Mexican stock market. The model is calibrated with data from the Central Securities Depository in Mexico from 2007-2010. Unlike classic models, the proposed model captures the interrelationship between risk factors (cause-effect), enhancing its profitability. Meanwhile, Davila et al. (2016) identify and quantify the operational risk factors associated with the electronic processing of financial transactions of a financial company. For the implementation of the model, events are simulated over six years, which allows them to model the cause-effect relations between different operational risk factors, and also incorporates elements that are not considered in the traditional model for calculating the OpVaR in the proposed model, providing better conditions of credibility to this value.

Martínez et al. (2016) developed a Bayesian methodology for identifying, quantifying, and measuring operational risk in several lines of business in commercial banking. The proposed model considers a priori and a posteriori distribution to estimate frequency and severity. The analysis is performed for the following lines of business: marketing and sales, retail banking, and private banking, which accounted for 88.5% of losses in 2011. Likewise, Dávila and Ortiz (2019), using the Solvency II regulatory framework, study the insurance sector, proposing a methodology for the identification and measurement of operational risk for the determination of the minimum required solvency capital for the policy quotation process for an insurance line, through a priori and a posteriori distribution with Monte Carlo simulation. Finally, Chen et al. (2021) apply the operational risk model to an offshore study.

Actuarial models are also considered in operational risk; for example, Lefevre and Picard (2021) use some basic elements of finite operator calculus to develop a new recursive method for calculating the probability distribution of a composite sum. They consider a convolution-type distribution for random count variables; actuarially, Panjer's algorithm is applied to collective, operational, and credit risk models. On the other hand, Nguyen et al. (2021) propose an operational risk analysis model with uncertainty quantification using three indicators: experts' ignorance, disagreement among experts, and polarization of their evaluations. The proposed model identifies critical and uncertain risks, and the study reveals physical flow as the dominant source of high-ranking risks with potentially significant consequences such as piracy, hazardous cargo, and maritime accidents. In operational and financial reporting, the risks are more uncertain, particularly cargo misreporting and unexpected fuel cost increases.

Regarding the Conditional Value at Risk (CVaR) actuarial model, Pinto et al. (2015) use this model to select structural trees with high returns and a wide range of risks, especially with low risk-aversion scenarios. However, as risk aversion increases, the pattern diversifies.

The assortment of methods for operational risk quantification is associated with varying levels of sophistication. According to the Basel Accords, the AMA corresponds to advanced measurement models that allow the flexible quantification of operational risk and help entities design their modeling and measurement system for this risk adapted to their particular circumstances. The Basel Accords do not determine a particular model but establish a set of qualitative and quantitative standards to be met by the internal model to be implemented. There are two essential standards: the measure of operational risk is an OpVaR (Operational Value at Risk) with a 99.9% confidence level, and the quantification method must incorporate potential severe loss events in the tail. The Operational Value at Risk at a confidence level α, denoted OpVaR (α), means the level of losses that are only exceeded with a probability 1 - α, over the period considered.

Methodology

Loss Distribution Approach (LDA)

Among the advanced AMA measurement methods, the most widely used in the literature is the loss distribution method, LDA. In this context, based on the studies by Franco and Velásquez (2010), Murillo et al. (2014) and Venegas et al. (2015), the quantification of operational risk through Monte Carlo simulation for a health service provider is studied in depth.

The implementation of an LDA includes the independent modeling of the severity probability distribution and the frequency probability distribution corresponding to the losses, to be subsequently combined, in this case through Monte Carlo simulation, in order to produce the aggregate loss distribution for each line of business/risk type combination for a specific period.

Historically, LDAs have been used in financial modeling for the insurance industry (Bühlmann, 1970). An LDA is intended to assess the operational risk for a company and its different business units. It is based on collecting information corresponding to internal historical losses (frequency and severity), which can be complemented with external data.

According to Basel, operational risk exposure is fragmented into a financial institution's lines of business and events. Specifically, eight lines of business are evaluated (corporate finance, trading and sales, retail banking, commercial banking, payments and settlement, agency services, asset management, and retail brokerage), and seven loss events are considered (external fraud, internal fraud, clients, process execution and management, technological failures, damage to physical assets, labor relations).

In order to implement the RMS in health service providers, losses caused by risk events that consider environmental, socioeconomic, or political factors are considered. It is necessary to consider aspects such as unforeseen increases in morbidity indices, variations in health conditions, changes in the population's behavior, or insufficient technical reserves. For a health service provider, the LDA model for the valuation of operational risk can be applied to make valuations at an aggregate level in the company. It can also be implemented individually in the various guidelines established by the entity or regulatory body.

The LDA model presents estimates for the aggregate loss by line of business and by event, which are combined to quantify the total operating loss. In the LDA, the total loss is established from the random sum of the various losses:

$$LDA = \sum_{k=1}^{7} \sum_{s=1}^{8} l_{ks}$$

(1)

where l_{ks} is the total loss in cell k, s of the loss matrix.

The l_{ks} are calculated as follows:

$$Z_{ks} = \sum_{F=1}^{n} M_{F_{ks}}$$

(2)

where Fks is the random variable representing the number of risk events in cell k,s (frequency of events) and MFks is the amount of loss in cell k,s (severity of the event). Therefore, losses correspond to the result of at least two different sources of randomness, frequency and severity.

The quantification of operational risk corresponding to cell k,s, will be represented by a given α percentile (e.g., 99.9%) of the aggregate loss distribution per period in that cell, which is simply denoted as S(m), given that the analysis is done initially for each specific cell. The distribution of S(m) is obtained by studying separately the loss frequency distribution pn = P(F = n) and the loss severity distribution fM(m). These two distributions are assumed to be independent and stable over time.

The distribution of aggregate losses corresponds to a composition between the discrete random variable associated with the frequency and the continuous random variable related to the severity of risk events. The aggregate loss distribution per period can be obtained as the weighted mean of the nth convolution of the severity, where the weights are the mass probabilities of the frequencies. The nth convolution of the severity is the probability of occurrence of the aggregate of n individual losses.

If the aggregate losses for the specific cell are given by S = M1 + M2 + ⋯ + MN = ∑Ni=1 Mi, where N is the event count random variable, and Mi is the random variable severity per event occurrence, and Mi are assumed independent and identically distributed with the common distribution function given by FM(m) = P(M ≤ m), then the nth convolution of the severity distribution, denoted by FM∗n(m) , is given by: P(M1 + M2 + ⋯ + Mn ≤ m) = F ∗ F ∗ ... ∗ ∗ F(m) = FM∗n(m), and therefore the aggregate loss distribution function is given by:

$$T_S(m) = P(S \leq m) = \sum_{m=0}^{\infty} P(M = m) F_M^{*n}(m)$$

(3)

Since there is no analytical alternative to express the composite loss distribution, it is necessary to apply numerical algorithms to calculate it. In the field of operational risk modeling, the most widely accepted methods are Monte Carlo simulation, Panjer's recursion algorithm, and inversion techniques using transforms; in addition, it is possible to use the closed-form approach of Böcker and Klüppelberg (2005), which is applicable for some specific circumstances, as discussed in Franco and Velásquez (2010). This article presents the theoretical background and application for two of these alternatives: the Monte Carlo simulation method and the so-called analytical approach of Böcker and Klüppelberg.

In the LDA in general, employing the aggregate loss function S(m) to be established, the capital charge for operational risk, OpVaR (α), as defined above, is established as:

I) $OpVaR(\alpha) = T_S^{-1}(\alpha)$, corresponding to a given confidence level α, which Basel has set at 99.9%, where T_S is the aggregate loss distribution function.

II) If the entity demonstrates the realization of provisions for expected losses, the capital charge is achieved by subtracting the expected losses from the α percentile. Thus: $OpVaR(\alpha) = T_S^{-1}(\alpha) - E(S)$. The OpVaR, with confidence level α, represents the level of operational risk losses that is only exceeded with a probability of $1 - \alpha$, over the period considered.

Frequency and severity distributions

For frequency modeling, a random variable F must be selected to represent the number of risk events per period in the cell (k,s) under consideration (frequency of events), and which follows a probability distribution $p(n) = P(F = n)$, $n \in [0,N]$. In this context, several authors, such as Cruz (2002), Gonzalez (2004), Shevchenko (2010), and Marshall (2001) show that the Poisson distribution fits many real operational risk situations. However, they also suggest considering alternatives such as Binomial or Negative Binomial.

Subsequently, it is necessary to find the best-fit probability distribution for the observed operational loss data for a given line of business and loss event. In this regard, Marshall (2001), Cruz (2002), González (2004), Venegas (2008), Shevchenko and Donnelly (2005), and Carrillo and Suárez (2004) propose the lognormal or Weibull distributions as the most recommendable alternatives for severity modeling. Thereby a random variable M is found, denoting the amount of loss in cell (k,s) of the loss matrix (severity or economic impact per event), with density function fM(m), and distribution function FM(m), given by $FM(m) = P(M \leq m)$. In order to capture potential extreme events, a heavy-tailed distribution could also be chosen to represent these potential loss amounts.

In the case of severity distributions, among the heavy-tailed distributions with support $(0, \infty)$, the distributions generally used in operational risk modeling are: the lognormal distribution, the generalized extreme value distribution, the generalized Pareto distribution, and the Weibull distribution. It is common in the literature to find that lognormal and Weibull distributions better fit operational loss data than most operational risk data. However, they exhibit relatively weak tail performance, given that operational losses tend to have heavier tails than these distributions, so large loss underestimates could result. In comparison, the Pareto distribution produces a good fit in the tail when there is sufficient data to allow this analysis but a weak fit in the body of the distribution. Klugman et al. (2004) present a thorough analysis of various statistical issues, hypothesis testing, and parameter estimation for loss models.

Modeling of aggregate losses

The following section presents the Monte Carlo simulation method to quantify the loss matrix's aggregate losses in a specific cell (k, s). The method is outlines, and the steps of the algorithm are established so that it can be replicated for application to any entity in the health sector with the corresponding information for operational risk modeling. The algorithm can be implemented in Matlab software for multiple frequency and severity distribution combinations.

Monte Carlo Simulation (MCS)

This method estimates aggregate loss distribution using several hypothetical scenarios randomly generated from the severity and frequency distributions. Chapelle et al. (2005) use the Monte Carlo simulation procedure by modeling the frequency distribution using a Poisson distribution with a parameter equal to the average number of losses observed during the entire period.

The procedure consists of the following steps:

- Determine the line/event combination to be simulated.
- Generate a random sample of the assumed frequency distribution.
- Generate a random sample of the severity distribution.
- Generate the aggregate loss distribution.

Following Venegas et al. (2015), the Monte Carlo simulation algorithm is developed as follows:

- Generate a random value n from the frequency distribution.

- Generate n random values of the severity distribution. Denote these losses simulated by L1, L2, ... Ln.

- Add the n simulated losses and obtain a loss for the period: S = L1 + L2 + ⋯ Ln.

- Return to step 1, and repeat a very large number M of times. Therefore, $S_1, S_2, … , S_M$ are obtained.

- Form the histogram of S_1, S_2, … , S_M, representing the simulated aggregate loss distribution for the period.

- Calculate the 99.9th percentile of the simulated aggregate loss distribution.

- Calculate the mean of the simulated aggregate loss distribution for the period. The expected loss (EL) is calculated as the mean of these simulated aggregate losses.

- The capital charge for operational risk, the OpVaR(99.9%), will be the 99.9th percentile of the aggregate simulated losses, or the difference between that percentile and the mean of the aggregate simulated loss distribution when provisions for expected losses are in place.

Closed-form approach of Böcker and Klüppelberg

An analytical alternative—in other words, one that admits a closed form for estimating operational VaR—is put forward by Böcker and Klüppelberg (2005), who researched a simple loss distribution model for operational risk, showing that one can have an approximation for OpVaR(α), in closed form, when the loss data are heavy-tailed, as occurs in many real-life situations.

The analytical approach proposed by Böcker and Klüppelberg (BK), as demonstrated below, is a direct formula, applicable in specific cases, for the quantile of the aggregate loss distribution in the form $G_S^{-1}(\alpha) = F^{-1}\left(1 - \frac{1-\alpha}{E(N)}\right)$, which directly relates the percentile $\alpha \to 1$, of the aggregate losses, to a high percentile given by $p = 1 - \frac{(1-\alpha)}{E(N)}$, of the simple distribution function of severities F, depending on the expected number of events per period $E(N)$.

This result, whose mathematical formulation is presented below, is based on an analytical property of distributions belonging to the class of subexponential distributions, which allows the

convolution to be expressed in the high severity limit as a function of the individual loss severity distribution.

The extreme value theory also supports the BK analytical approach for calculating the capital charge. According to one of the Basel requirements to incorporate the heavy-tailed properties of severity distributions, these authors consider the family of subexponential distributions, including lognormal distributions and distributions with heavier tails.

The Basel Accords, in the context of the AMA models, allow financial institutions the flexibility to implement their internal model and operational risk quantification system. Instead of prescribing a specific model, qualitative and quantitative standards that the internal models must meet are proposed. Among the most significant quantitative criteria, and the basis of the analytical approach, is that the measure of operational risk is a VaR with a confidence level of 99.9%. In addition, the measurement method must capture potentially severe loss events in the tail of the loss distribution.

The basic theoretical elements and the mathematical support of the analytical approach of the BK model are presented below.

Sub-exponential severity distributions

To consider the heavy-tailed property of the severity distributions, which is a requirement of Basel II, the best-known distributions with that property are considered, which are lognormal, Weibull, and Pareto, which belong to the so-called class of subexponential distributions with support $(0, \infty)$, and are described in Table 1.

Table 1
Some Sub-exponential Distributions

Name	Distribution function	Parameters
Lognormal	$F(x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$	$\mu \epsilon R, \sigma > 0$
Weibull	$F(x) = 1 - e^{-\left(x/\theta\right)^{\tau}}$	$\theta > 0, 0 < \tau < 1$
Pareto	$F(x) = 1 - \left(1 + \frac{x}{\theta}\right)^{-a}$	$a, \theta > 0$

Source: Böcker and Klüppelberg (2005)

The essential characteristic of subexponential distributions is that their tails decay more smoothly than any exponential tail. In other words, the tail of the sum of n subexponential random variables has the same order of magnitude as the tail of the maximum variable. In analytical form, it follows that:

$$\lim_{x \to \infty} \frac{P(X_1 + X_2 + \cdots + X_n > x)}{P(\max (X_1, X_2, \ldots, X_n > x)} = 1, \text{ for some } n \geq 2$$

(4)

This means that the sum of n independent and identically distributed severities is more likely to be large because one of its terms is large; or in the case of operational risk, severe total losses are mainly caused by a single large loss rather than the consequence of accumulated small independent losses.

The standard LDA model has the following features:

a. The severities $\{X_i\}_{i \in \mathbb{N}}$ are positive, independent, and identically distributed random variables.

b. The number of loss events in an interval [0, t] , for t > 0, is random, and the corresponding frequency process is $\{N(t)\}_{t \geq 0}$.

c. The severity process and the frequency process are assumed to be independent.

d. The aggregate loss S(t) up to time t is given by $S(t) = \sum_{i=0}^{N(t)} X_i, t \geq 0$. Furthermore, if G_t is the aggregate loss distribution, then the Value at Operational Risk in period t, at confidence level α is given by $\text{OpVaR}_t(\alpha) = G_t^{\leftarrow}(\alpha) = \inf \{x \in R : G_t(x) \geq \alpha\}$, with $0 \leq \alpha \leq 1$. In particular, if G_t is strictly increasing and continuous, then $\text{OpVaR}_t(\alpha) = G_t^{-1}(\alpha)$.

The essential mathematical foundations of the BK approach are presented below, without the demonstrations, which can be found in Franco and Velásquez (2010).

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If F is a subexponential distribution, then given ε > 0, there exists a finite constant K such that, for all $n \geq 2, \frac{\overline{F^{n*}(x)}}{\overline{F}(x)} \leq K(1 + \varepsilon)^n, x \geq 0$.

EKM Theorem (Embrechts, Klüppelberg, & Mikosch)

Let the standard LDA $S(t) = \sum_{i=0}^{N(t)} X_i, t \geq 0$. Assume that the severities X_i are subexponential with distribution function F. Set $t > 0$ and define the frequency distribution by $P(N(t) = n) = p_t(n)$, for $n \in N_0 = N \cup \{0\}$, and the aggregate loss distribution given by $G_t(x) = \sum_{n=0}^{\infty} p_t(n) F^{n*}(x), x \geq 0, t \geq 0$, where $F(.) = P(X_k \leq .)$ is the distribution function of X_k and $F^{n*}(.) = P(\sum_{i=1}^{n} X_i \leq .)$ is the nth convolution of F with $F^{1*} = F$ y $F^{0*} = I_{[0,\infty)}$.

If for some $\varepsilon > 0$, $\sum_{n=0}^{\infty} (1 + \varepsilon)^n p_t(n) < \infty$, then it is concluded that $\lim_{x \to \infty} \frac{EN(t)\overline{F}(x)}{\overline{G_t}(x)} = 1$. Or equivalently:

$$\overline{G_t}(x) \sim EN(t)\bar{F}(x), \; x \to \infty$$

(5)

Where

$EN(t)$ is the expected value of the loss frequency.

$\bar{F}(x) = 1 - F(x)$ is the tail severity distribution, y $\overline{G_t}(x) = 1 - G_t(x)$ is the aggregate tail loss distribution

The theorem of the analytical formula for OpVaR

Consider the standard LDA model for a fixed t > 0 and a subexponential severity with distribution function F. Suppose, in addition, that the tail estimate (5) is satisfied. Then:

$$OpVaR_t(\alpha) = F_t^{\leftarrow}\left(1 - \frac{1-\alpha}{EN(t)}(1 + o(1)\right), \text{when } \alpha \to 1$$

(6)

Franco and Velásquez (2010) present some essential characteristics of this approach:

a   For high confidence levels, the OpVaR depends only on the tail, not the body, of the severity distribution. Therefore, if the objective is to calculate the operational VaR, it is unnecessary to model the entire distribution function F.

b   The frequency distribution only intervenes with its expected value in expression (6). Therefore, to apply this model, it would be sufficient to estimate the sample mean of the frequencies. This implies that the overdispersion of a model such as the negative binomial distribution would asymptotically have no impact on OpVaR.

c   Since the capital charge is based on a very high quantile of the aggregate loss distribution G_t, it is natural to estimate OpVaR using the asymptotic behavior of the tail and quantile estimate. Instead of considering the entire distribution, it is sufficient to concentrate on the right tail P(S(t) > x) for a large value of x.

Applying Equation (6) to the most common subexponential distribution functions, closed-form solutions for an asymptotic $OpVaR_t(\alpha)(\alpha \to 1)$ are immediately obtained for the corresponding severity distributions, as shown in Table 2.

Table 2
Asymptotic approximations for OpVaR$_t(\alpha)$, for the most common severity distributions

Distribution	$OpVaR_t(\alpha)$
Lognormal	$\exp\left[\mu - \sigma\emptyset^{-1}\left(\dfrac{1-\alpha}{EN(t)}\right)\right]$
Weibull	$\theta\left[ln\left(\dfrac{EN(t)}{1-\alpha}\right)\right]^{1/\tau}$
Pareto	$\theta\left[\left(\dfrac{EN(t)}{1-\alpha}\right)^{1/a} - 1\right]$

Source: Böcker and Klüppelberg (2005)

Where EN(t) is the expected value of the frequency distribution and $\emptyset$ is the standard normal distribution function.

Application for a health service provider in Colombia

An HSP operating in Colombia provided for this study a series of data on process failures, which led to economic losses during 2019 and 2020, which can be considered adverse events. The analyzed HSP covers the entire national territory of Colombia, currently has more than 200 thousand affiliates, and has assets above COP 350 billion. The frequency and severity data corresponding to losses due to the materialization of operational risks for the years 2019 and 2020 are presented in Table 3.

Table 3
Operational risk failures 2019-2020

Month	Frequency 2019	Severity (in millions COP)	Frequency 2020	Severity (in millions COP)
1	5	242.8	3	352.7
2	7	2194.5	9	720.6
3	4	1316.8	8	1381.5
4	6	680.3	11	2293.3
5	7	325.7	14	1874.7
6	5	406.9	10	438.2
7	7	3427.2	14	3578.3
8	6	1285.6	8	262.5
9	13	473.1	11	1428.5
10	8	1289.8	12	1352.4
11	13	1363.9	8	504.0
12	8	1793.0	10	1350.1

Source: created by the author based on data from the analyzed HSP

The severity recorded in each month corresponds to the sum of the losses for the month. Nevertheless, distribution fits for severity are made with individual losses.

For eventual coverage of the potential loss from operational risk for these years, the entity maintained provisions per period of COP 6500 million and COP 6750 million, respectively.

In 2019, a mean of 7.42 occurrences per month for the observed frequencies existed, with a standard deviation of 2.87. The mean was COP 1233.31 million per event for the individual severities observed, with a standard deviation of COP 926.97 million. Concerning the year 2020, for the observed frequencies there is a mean of 9.83 occurrences per month, with a standard deviation of 3.01; and for the observed severities there is a mean of COP 1294.73 million per occurrence, with a standard deviation of COP 963.96 million.

Distribution fit for frequency 2019

Regarding the distribution fit for Frequency, using the Risk Simulator software it was found that the three distributions that best fit the observed Frequency data, in order from best to least fit, were, respectively, Poisson, binomial, and negative binomial. The essential characteristics of these fitted distributions are presented in Table 4 below.

Table 4
Distributions fitted for Frequency 2019

	First choice			Second choice		Third choice
	Fitted Distribution	Poisson	Fitted Distribution	Binomial	Fitted Distribution	Negative Binomial
	Lambda	8.44	Tests	62	Required Successes	65
			Probability	0.13	Probability	0.9
	Chi-Square Statistics	14.08		15.12		17.18
	Real	Theoretical		Theoretical		Theoretical
Mean	7.42	8.44		8.34		7.44
Standard Deviation	2.87	2.90		2.69		2.88
Skewness	1.26	0.34		0.27		0.43
Kurtosis	0.94	0.12		0.04		0.21

Source: created by the author using Risk Simulator

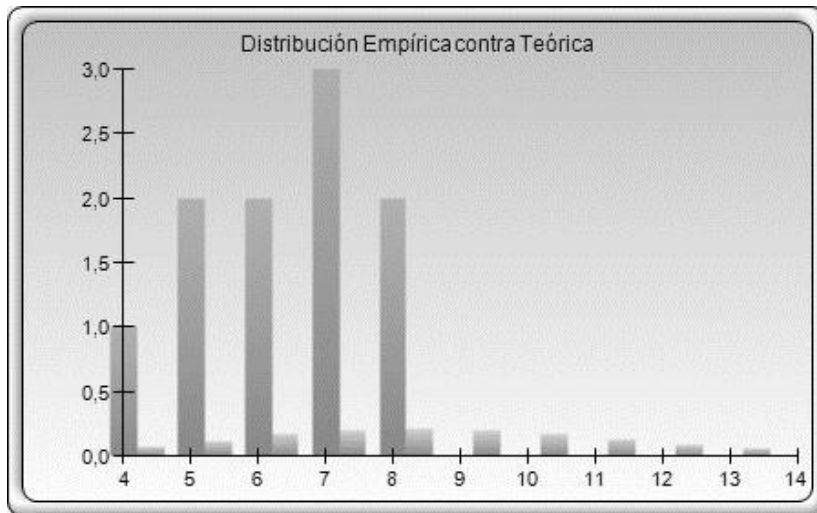The graph corresponding to the Poisson distribution fit is presented in Figure 1 below.

Figure 1. Result of the Poisson distribution fit for frequency
Source: created by the author using Risk Simulator

The three distributions considered are very common in operational risk for modeling the discrete variable frequency. Based on the hierarchy of these distributions as appropriate settings for the discrete variable Frequency, for the application of the Monte Carlo simulation, the Poisson distribution with a mean of 8.44 and standard deviation of 2.90 is chosen, as shown in Table 4.

Distribution fit for severity-2019

Considering the most usual distributions in operational risk, and using the Risk Simulator software, it was found that the three distributions that best fit the individual severity data, in order of greatest to least fit, were, respectively, the lognormal, Gumbel Maximum, and Exponential distributions. The essential characteristics of these fitted distributions are presented in Table 5 below.

Table 5
Distribution fits for Severity 2019

		First choice		Second choice	Third choice	Third choice
	Fitted Distribution	Lognormal	Fitted Distribution	Gumbel Maxima	Fitted Distribution	Exponential
	Mean	153.89	Alpha	105.49	Location	43.55
	State Dev.	108.77	Beta	70.69	Lambda	0.01
	Kolmogorov Smirnov statistic	0.04		0.06		0.06
	Statistic Test for P-Value	0.9998		0.9020		0.8817
	Real	Theoretical		Theoretical		Theoretical
Mean	156.03	153.89		146.30		160.91
Standard Deviation	112.47	108.77		90.66		117.36
Skewness	1.85	2.47		1.14		2.00

Source: created by the author using Risk Simulator

The graph corresponding to the lognormal distribution fit is presented in Figure 2.
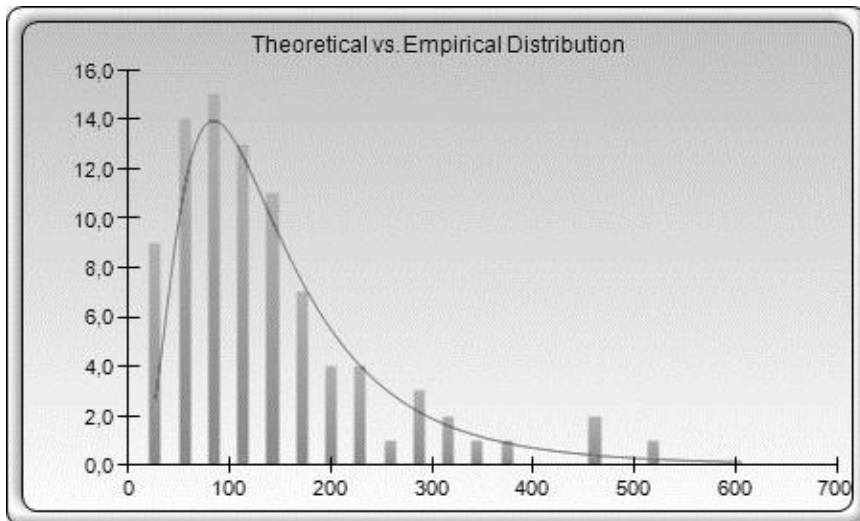

Figure 2. Distribution Fit Graph for Severity 2019
Source: created by the author using Risk Simulator

The three distributions obtained are common in operational risk for modeling the continuous loss default variable. Based on ranking these distributions as appropriate fits for the continuous variable Severity, the lognormal distribution with mean COP 153.89 million and standard deviation COP 108.77 million would be chosen for the Monte Carlo simulation application, as shown in Table 5.

In conclusion, to generate the aggregate loss distribution in this context, a convolution must be made between a Poisson distribution with a mean of 8.44 and a lognormal distribution with a mean of COP 153.89 million and a standard deviation of COP 108.77 million.

Aggregate loss distribution by frequency and severity convolution 2019

In this section, the previously described Monte Carlo simulation algorithm is applied to perform the convolution of the selected frequency and severity distributions, generate the aggregate loss distribution, and hence calculate the operational VaR. Specialized Matlab software was used for this purpose.

Applying this algorithm, 1000000 iterations were performed, and the aggregate loss distribution shown in Figure 3 was found.
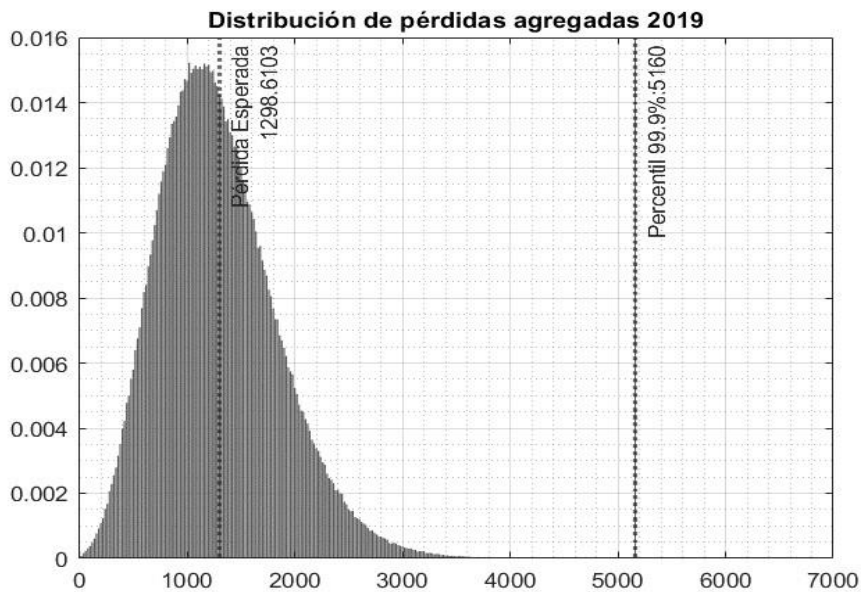

Figure 3. Aggregate loss distribution chart 2019
Source: created by the author using Matlab

In this simulated distribution of aggregate losses, there is a mean of approximately COP 1298.61 million, corresponding to the so-called expected losses, and a standard deviation of COP 549.59. Additionally, the 99.9% percentile is COP 5160 million.

According to these results, and with the theory proposed, the monthly operational VaR for a confidence level of 99.9%, denoted OpVaR(99.9%), is COP 5160 million, under the assumption that no provisions are made. Moreover, in this case, when provisions are made for expected losses, the capital charge for operational risk, which is the differential between the 99.9% percentile and the expected losses, as previously defined, would have a value of approximately COP 3861.39 million per month.

Distribution fit for frequency for 2020

Concerning the distribution fit for Frequency, using the Risk Simulator software it was found that the three distributions that best fit the observed Frequency data, in order from best to least fit, were, respectively, Poisson, binomial and geometric. The essential characteristics of these fitted distributions are presented in Table 6.

Table 6
Distributions fitted for Frequency 2020

	First choice				Second choice	Third choice
	Fitted Distribution	Poisson	Fitted Distribution	Binomial	Fitted Distribution	Geometric
	Lambda	9.31	Tests Probability	63 0.14	Probability	0.1
	Chi-Square Statistics	17.13	Chi-Square Statistics	19.82	Chi-Square Statistics	40.51
	Real	Theoretical		Theoretical		Theoretical
Mean	9.83	9.31		9.05		9.48
Standard Deviation	3.01	3.05		2.78		9.97
Skewness	-0.72	0.33		0.26		2
Kurtosis	1.42	0.11		0.03		6.01

Source: created by the author using Risk Simulator

The graph corresponding to the Poisson distribution fit is presented in Figure 4.
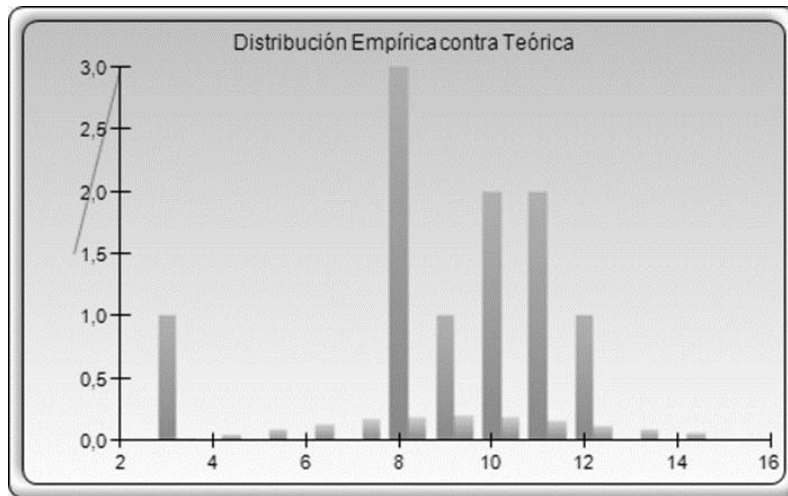

Figure 4. Distribution Fit Graph for Frequency 2020
Source: created by the author using Risk Simulator

The three distributions considered are very common for modeling the discrete variable frequency in operational risk. Based on the hierarchy of these distributions as appropriate settings for the discrete variable Frequency, for the application of the Monte Carlo simulation, the Poisson distribution with a mean of 9.31 and standard deviation of 3.05 is chosen, as shown in Table 6.

Distribution fit for severity 2020

As for distribution fit for severity, using the Risk Simulator software it was found that the three distributions that best fit the observed severity data, in order from best to worst fit, were, respectively, the lognormal, Gumbel Maximum, and Gamma distributions. The essential characteristics of these fitted distributions are presented in Table 7 below.

Table 7
Distribution fits for Severity 2020

	First choice		Second choice		Third choice	
	Fitted Distribution	Lognormal	Fitted Distribution	Gumbel Maxima	Fitted Distribution	Gamma
	Mean	212.22	Alpha	149.34	Alpha	3.15
	State Dev.	138.05	Beta	80.16	Beta	64.32
	Kolmogorov Smirnov statistic	0.04		0.05		0.05
	Statistic Test for P-Value	0.9972		0.9353		0.9232
	Real	Theoretical		Theoretical		Theoretical
Mean	204.36	212.22		195.61		202.86
Standard Deviation	114.65	138.05		102.81		114.23
Skewness	1.19	2.23		1.14		1.13

Source: created by the author using Risk Simulator

The graph corresponding to the lognormal distribution fit is presented in Figure 5.
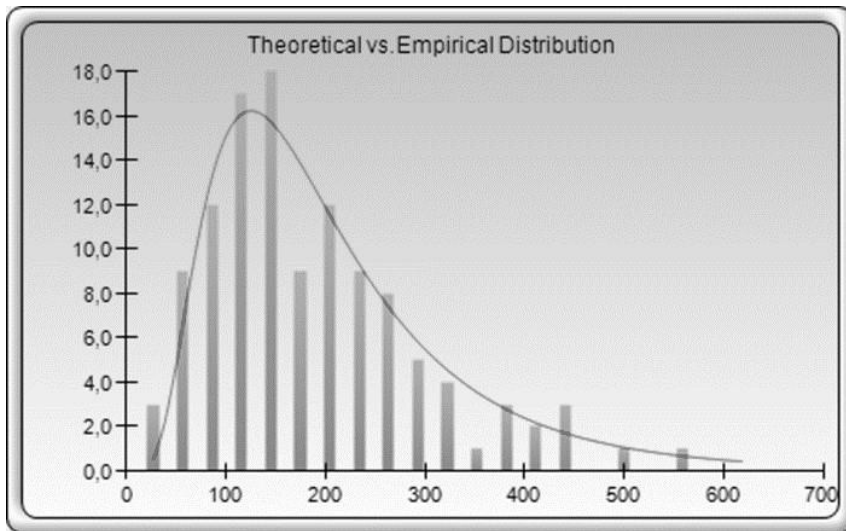

Figure 5. Distribution Fit Graph for Severity 2020
Source: created by the author using Risk Simulator

The three distributions obtained are very common in operational risk for modeling the continuous loss default variable. Based on the ranking of these distributions as appropriate fit for the

continuous variable Severity, for the application of the Monte Carlo simulation, the lognormal distribution with mean COP 212.22 million and standard deviation COP 138.95 million is chosen, as shown in Table 7.

In conclusion, to generate the aggregate loss distribution in this context a convolution between a Poisson distribution with a mean of 9.31 and a lognormal distribution with a mean of 212.22 million and a standard deviation of COP 138.952 million must be performed.

Aggregate loss distribution by frequency and severity convolution 2020

In this section, the Monte Carlo simulation algorithm previously described is applied to perform the convolution of the selected frequency and severity distributions and generate the aggregate loss distribution, from which the operational VaR is calculated. Specialized Matlab software will be used for this purpose.

1000000 iterations were performed applying this algorithm, and the aggregate loss distribution shown in Figure 6 was found.
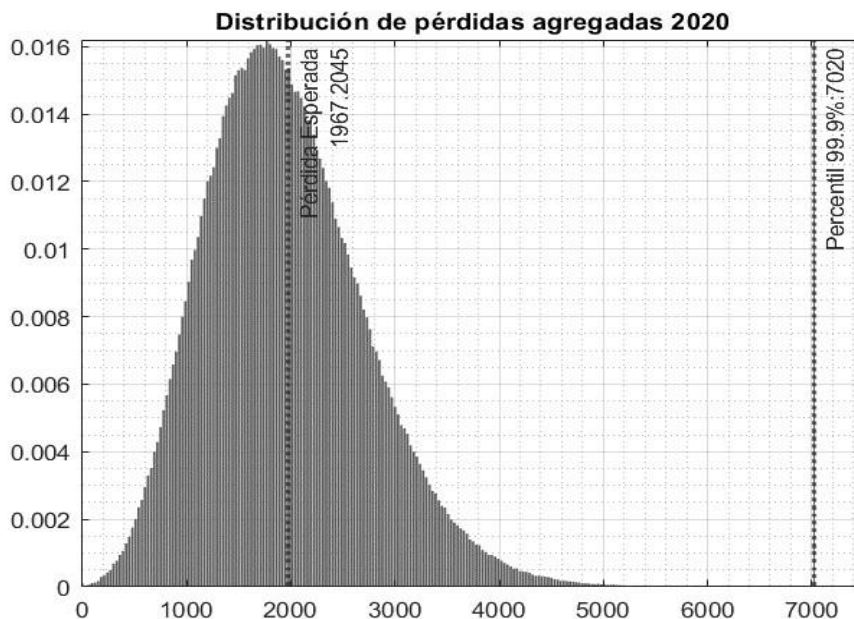

Figure 6. Aggregate loss distribution chart 2020
Source: created by the author using Matlab

In this simulated distribution of aggregate losses, there is a mean of approximately COP 1967.20 million, corresponding to the so-called expected losses, and a standard deviation of COP 767.56 million. Additionally, the 99.9% percentile is COP 7020 million.

According to these results, and with the theory proposed, the operational VaR for a confidence level of 99.9%, which is denoted OpVaR(99.9%), is given by COP 7020 million under the assumption that no provisions are made. Furthermore, since there are provisions for expected losses in this case, the capital charge for operational risk, which is the differential between the 99.9% percentile and expected losses, as previously defined, would have a value of approximately COP 5052.8 million.

Comparative analysis of results obtained through MCS

Table 8
MCS Synthesis 2019-2020

	Year 2019			
	Frequency	Fitted Severity	Aggregate losses	OPVaR(99.9%) (COP millions)
Mean	8.44	153.89	1298.61	
Standard deviation	2.90	108.77	549.59	
99.9% percentile			5160	3861.39
	Year 2020			
Mean	9.31	212.22	1967.20	
Standard deviation	3.05	138.05	767.56	5052.80
99.9% percentile			7020	5052.80

Source: created by the author using Risk Simulator

According to the best distribution fits found and the estimates generated through MSMC, there is an increase in the frequencies, in which the average increases by 10.31% from 2019 to 2020. In addition, there was a much more noticeable increase in the behavior of severities, in which the average increased by 37.9%. These results increase monthly OpVaR from COP 3861.39 to COP 5052.80 million, representing a growth of 30.85% from 2019 to 2020, which is considered a very steep increase from one year to the next. This significant increase can be attributed to the effect of the pandemic. Still, it is also essential to remember that in many real-life situations related to operational risk, losses materialize not only months but years after the event, and therefore may not all be reflected in the data considered.

The Böcker and Klüppelberg analytical approach

In order to illustrate the benefits of this proposed alternative, the Böcker and Klüppelberg approach is applied, assuming that a good fit for severity is the lognormal distribution with the indicated parameters, which is subexponential, and which is one of the assumptions of the approximation. This method is an option for Op-VaR quantification when all assumptions are satisfied.

Consequently, an analytical approach for 2019 was applied, assuming a frequency-severity combination using Poisson distributions with mean EN(t) = 8.44, and lognormal severity with a mean of 153.89 and standard deviation of 108.77. Similarly, for the year 2020, the analysis used Poisson distributions with mean EN(t) = 9.31, and lognormal severity with mean 212.22 and standard deviation 138.05.

Recalling that for a random variable X with lognormal distribution, it is concluded that $E(X) = e^{\mu+\frac{\sigma^2}{2}}$ and $V(X) = \left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}$, then μ = 5.83 and σ = 0.64 for the year 2019 are obtained, and similarly, μ = 6.28 and σ = 0.59 for the year 2020.

Hence with the Böcker and Klüppelberg Analytical Approach, applied to the year 2019, EN(t) = λ = 8.44 for the Poisson distribution, and the lognormal μ = 5.83 and σ = 0.83. The corresponding values for the year 2020 would be EN(t) = λ = 9.31 for the Poisson distribution, and for the lognormal μ = 6.28 and σ = 0.59.

For different confidence levels the results are shown in Table 9.

Table 9
Application of the Böcker and Klüppelberg analytical approach

Confidence Level (α)	2019		2020	
	OpVaR		OpVar	
	$\phi^{-1}\left(\dfrac{1-\alpha}{EN}\right)$	$OpVaR = \exp\left(\mu - \sigma\phi^{-1}\left(\dfrac{1-\alpha}{EN}\right)\right)$	$\phi^{-1}\left(\dfrac{1-\alpha}{EN}\right)$	$OpVaR = \exp\left(\mu - \sigma\phi^{-1}\left(\dfrac{1-\alpha}{EN}\right)\right)$
95%	-2.516629	1689.06	-2.551010	2429.37
99%	-3.039506	2356.09	-3.068936	3304.54
99.9%=OpVaR	-3.675949	3532.91	-3.700916	4810.09

Source: created by the author

Applying this method, with the assumptions made, it was found that the monthly OpVaR(99.9%) for 2019 would be COP 3532.91 million, and for 2020 it would be COP 4810.09 million. This approach would result in an increase of 36.15 % in the estimated operational risk.

The significant differences between the OpVaR for different confidence levels show that the method is very sensitive to the behavior of severities in the tail of the distribution, as proposed in the theory.

Although the Böcker and Klüppelberg approach generated a result relatively close to that obtained through SMC, it is essential to emphasize that proper enterprise risk management implies continuously applying different methodologies and evaluating their performance. This analytical approach is a valid alternative for quantifying operational risk when the assumptions are satisfied.

Conclusion

Quantification of operational risk is required to comply with regulatory requirements. However, circumstances such as the Covid-19 pandemic have materialized the health sector's vulnerabilities to adverse events. They have also demonstrated the need to strengthen risk management systems (RMS) for the health sector worldwide to contribute to its sustainability. In addition, during the pandemic, the importance of HSP stability in the economic performance of surrounding communities has been demonstrated. There is an aggravating factor that, unlike entities in other sectors, such as the financial sector, the materialization of operational risks in the health sector not only generates potential losses in economic terms, but the consequences can translate into the loss of human lives, caused by the materialization of adverse events, in many cases avoidable, due to failures in the systems, people, and processes of the HSP.

The advanced operational risk measurement methods applicable to the healthcare sector, such as the Monte Carlo simulation approach and the Böcker and Klüppelberg analytical approach applied in this article, can be replicated and implemented as an internal advanced measurement method to strengthen the quantification of operational risk in any healthcare entity worldwide that has records of adverse events and their respective economic consequences. This alternative can generate an estimate of capital charges more in line with the particular circumstances of each entity since, regardless of the standard regulations of each country on operational risk, each entity can implement internal systems for risk management, which can be validated by the regulator, and which incorporate specific internal characteristics.

The Covid19 pandemic has subjected HSPs to maximum levels of performance and stress on the part of their personnel, systems and processes. This is in an environment of limited resources and additional factors, such as the transformation toward virtual healthcare. Operational risk has been significantly affected in the health sector entities, demonstrating the need to strengthen the ORMS significantly. This article presents two alternatives for advanced quantification of operational risk applicable to the health sector. Regarding the monthly operational risk measurement results in the case

study analyzed, there was a significant increase from 2019 to 2020. In addition, it is essential to consider that losses materialize with delays in many real-life situations related to operational risk, so they may not all be reflected in the data considered and may impact the results of subsequent years.

Capital charges for all types of risk, including operational risk, are provisions or reserves that ultimately require capital to be tied up to some extent. It is recommended that healthcare entities use multiple alternatives for the modeling and quantification of operational risk so that they can strengthen their RMSs, evaluate the results comparatively and strategically define the model to be used and presented to regulatory bodies.

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