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Competition in the Mexican credit system: A biological approach

Competencia en el sistema crediticio mexicano: una aproximación biológica

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Abstract

The speed and adaptation of Mexican banks that grant loans, in the face of changes in the strategy of their competitors, with differential equation systems for an evolutionary game of "rock, paper or scissors" are analyzed; for the degree of competition, a mutation parameter of market strategies is included. The results show that for stable solution trajectories there must be an adaptation of strategies in the competitors; the non-zero mutation parameter results in a hidden variable that measures the degree of competition and governs the adaptation time temporal variable. The main contribution is the inclusion of a Brownian movement to the equation that models the payment function of each bank and a mutation factor to model the effect of asymmetric information, which allows measuring the degree of industrial competition. A limitation is not to consider the cases in which the degree of innovation is small or large.

JEL Code: C02, C63, C73, H81
Keywords: mathematics methods; simulation models, stochastics and dynamic games; credits

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Resumen

Se analiza la velocidad y adaptación de bancos mexicanos que otorgan créditos, ante cambios de estrategia de sus competidores, con sistemas de ecuaciones diferenciales para un juego evolutivo "piedra, papel o tijera"; para el grado de competencia, se incluye un parámetro de mutación de estrategias del mercado. Los resultados muestran que para trayectorias de solución estables debe existir adaptación de estrategias en los competidores; el parámetro de mutación no cero, resulta una variable oculta que mide grado de competencia y rige la variable temporal tiempo de adaptación. La principal aportación es la inclusión de un movimiento browniano a la ecuación que modela la función de pagos de cada banco y un factor de mutación para modelar el efecto de la información asimétrica, lo que permite medir el grado de competencia industrial. Una limitación es no considerar los casos en que el grado de innovación sea pequeña o grande.

Código JEL: C02, C63, C73, H81
Palabras clave: métodos matemáticos; métodos de simulación; juegos dinámicos y estocásticos; créditos

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Introduction

Economists have been trying for years to predict the behavior of rational economic agents seeking to maximize profits. To this end, industrial analysis and economics have used two main currents: the structuralist one and the one based on game theory.

This work uses the technology provided by game theory, relaxing the assumption of perfect rationality, which implies that economic agents may hesitate or imperfectly copy strategies known to be successful. In the case of this work, contamination by a Brownian diffusion process is established, which seeks to obscure the process of perfect economic rationality proposed by game theory.

While analyzing the problem of collusion within the industrial organization, there is the possibility of analyzing an environment in which market participants—hereafter referred to as players—can decide whether to cooperate or compete. If they decide to compete, it is possible to imitate other players' winning strategies.

Adapting this concept to the present work, it is proposed that the credit card market (consumer credit) is an industry in which players tend to imitate the strategies of other players when these turn out to be winners, given the low cost of this imitation. Although these are completely different scenarios, the metaphor used was that of the well-known rock-paper-scissors game, which represents an unstable equilibrium dependent on the response of the other player, with the possible strategies that an oligopolistic market may present when there is some degree of competition among them.

Changes of promotional strategies in the mass credit market, whether customer benefits, advertising strategy or intermediation margins, are easily imitated once they have gone to market.

Similarly, an attempt at differentiation by any of these means can be responded to either by imitation or differentiated response in a relatively quick and low-cost manner. This is evidently reminiscent of the rock-paper-scissors game.

The imitation capacity within the market is shown throughout this paper. Then, there is an analysis of the game in which the payoff functions lead to unstable equilibria that depend on the response of the other player if there is no strategy mutation parameter or equilibrium when the mutation is sufficiently small.

At this point, another of the key concepts of the work appears: the possibility of agents mutating, i.e., changing their strategy according to the payments they receive at each step of the repeated game, as mentioned by Mobilia (2010). This concept clearly recalls any species' evolutionary adaptations in the game of life. In particular, the analysis of evolutionary competition was based on the article by Sinervo (1996), where he describes a game similar to the rock-paper-scissors game played by lizards to achieve mating.

This is not a minor issue since the technology already constituted from the mathematical modeling of the reproduction of these reptiles is used first to analyze the competition of banks and then, through mutation, to obscure the supposed perfect rationality of all economic agents.

For example, a bank does not know the strategy of the rest exactly; nonetheless, it is capable of seeing the results and imitating them (even imperfectly). The costs of extracting the missing information and the ability to imitate are the factors that govern the speed of mutation of each bank's strategy, which implies that a rapid adaptation of strategy implies low barriers to the entry of that strategy (competition). Conversely, low imitation implies niche behavior (collusion) because banks choose not to compete with each other.

In summary, this paper combines the use of rock-paper-scissors games that seek to explain the reproductive strategy of lizards with a Brownian motion that obscures the perfect rationality enjoyed by economic agents in most of the current modeling. The use of the Brownian motion implies an unawareness of some elements of decision making that must be corrected iteratively on the fly, and that evidently change the strategy of all those involved in the game, thus accepting the response functions and outcomes of the game.

The paper will begin with a brief description of the elements of competition in the mass consumer credit market faced by the largest banking institutions in the country, analyzing the problem in light of a repeated game previously used in microeconomic analysis.

Subsequently, it will be shown that imitating other players' strategies is reasonable in this industry, given the low costs of strategy change and the imperfect information available to each player once the strategy is made public. The similarity that microeconomic behavior has with the reproductive behavior of lizards will also become apparent. In other words, the clear link in competition in which imitation is cheap will be established using a biological metaphor that has already been studied.

Finally, it will be established that the rock-paper-scissors game corresponds closely to the possible strategies and payoff functions available to players of both approaches.

Once the system is established, the rationality of economic agents under traditional game theory analysis will be obscured by the "fog" of uncertainty and imperfect knowledge obtained from public data once the trading strategy is developed. This paper analyzes the effect of the strategy mutation factor combined with the effect of the "fog" of imperfect knowledge on economic agents.

Conclusions are shown in which it is evident that banking institutions imitate within 3 to 6 months the successful strategies of their counterparts and that there is an apparent short-term equilibrium within the industry.

The paper is divided into seven sections. The first section contains the background of the research, the second the objective, the third describes the classification of possible collusions within an oligopoly, the fourth section deals with the Mexican credit portfolio, the fifth contains information on the methodology, the sixth section corresponds to the application of the model in the credit system, and then the conclusions, scopes, and limitations of the model are shown.

Background

Evolutionary games have been used to analyze various behavioral environments by authors such as Hosseini-Motlagh, Johari, and Zirakpourdehkordi (2020). They proposed an evolutionary game model to analyze the behavior of a population of financially constrained producers in Iran who obtain financial support from a dominant distributor as a function of their long-term investment. Huang and Zhu (2019) proposed a dynamic game with incomplete information to model a long-term interaction between a stealthy attacker and a proactive defender in cyber-physical security.

Works such as that of Diaz, Sosa, and Cabello (2019) search for the factors that influence household over-indebtedness using neural networks; they determine that the main factor is the existence of a bank loan. Hao and Gao (2019) use game theory to establish the private financial lending model constructed according to the potential restrictions between the subject and the object in the financial demand relationship.

Moreover, studies such as Prasanna and Sujit (2019) create a model using network analysis to understand systemic risk and guide the design of financial regulation. They analyze how network models and those based on epidemiological approaches provide a compelling description of the structure of realworld financial systems and of the different contagion mechanisms observed during the global financial crisis.

Articles such as Hofbauer and Sigmund (1998) or Toupo and Strogatz (2015) show the development of the model; there are also applications in local dispersion, as in Frean and Abraham (2001). Other studies where a similar environment is analyzed are those of dynamic games, as in Vasal, Sinha, and Anastasopoulos (2019) and Gensbittel and Grün (2018), while others analyze organisms that have similar behavior but allow changes in the strategy of the game, to obtain a higher probability of winning, Kang et al. (2013).

The 'rock-paper-scissors' game, known as such in English-speaking countries (Kodama et al., 2016), where the rock has an advantage over the scissors, the scissors have an advantage over the paper, and the paper over the rock is also known as janken in Japan, piedra, papel, y tijera in some Latin American countries, and even yaquenpoh in Chile, Peru, and Brazil. The first name will be used in this paper.

This game has been modeled and applied to biological systems, such as its application to the lizard Uta stansburiana described in Sinervo and Curt (1996), where there are three types of lizards: one is aggressive and covers large territories; another is less aggressive and covers small territories; and the third is not aggressive, but may appear to be female, which facilitates interaction. Individuals apply one of these three possible strategies and compete to mate, which generates an oscillating cycle in the number of specimens using each strategy.

Another example worth mentioning is that of Kerr et al. (2002), where the behavior of Escherichia coli bacteria is analyzed, which can present different reproduction strategies, leading to similar games and equilibria. Similar studies are those of Kirkup and Riley (2004) and Cameron, Whyte, and Antonovics (2009).

The analysis to be carried out is similar to that of Hofbauer and Sigmund (1998), but in this case, it is applied to the "rock-paper-scissors" game, which assumes that there is no dominant strategy and that each player can choose indifferently (without costs) any of the three strategies.

Aim

This paper aims to analyze the speed and type of adaptation of Mexican banks to changes in their competitors' lending strategies. This goal will be achieved through differential equation simulations with an embedded stochastic process.

Competition among companies

When there is perfect competition, producers are price takers; when this does not occur, only some competitors determine the price and then they have market power. This structure can be defined as an intermediate between monopoly and oligopoly. According to economic theory, when market power falls to one company, it is said that there is a monopoly; when there are two, a duopoly, and when there are more, it is called an oligopoly (Mas-Collel, Whinston, and Green, 1995).

A company is defined as dominant when it can act without considering its rivals or consumers. In an oligopoly, this power lies with a larger set of two companies capable of moving the market price by employing an agreement between them (collusion). This type of organization allows the company to take advantage of its position to exploit the market and thus maintain that position (Cuervo and García, 2004). An example of this phenomenon of asymmetric information is the deficient production linkage of MIPYMES (MSMEs). In this ecosystem, the large producer knows and can modify the prices of the target market or the supply chain given its privileged access due to brand power or market share, while small companies are subordinated to the choices of the large company; for more details see Zevallos and Vallejo (2003).

This paper distinguishes between three different types of cooperation within an oligopoly, namely:

- Cartel: described in Schmalensee (1978), Scherer (2002), Harrington (2005),
- Holding: described in Kamien and Zang (1990), and
- Trust: described in Burns (1986), Wilkinson and Burchell (1996)

According to Sabino (1991), a cartel is a group of companies concentrated in an area where they maintain agreements on the quantity and price of the good supplied in that sector or geographical area. A holding is defined as a business association in which shares are exchanged, while in a trust, there is total control of one company (subsidiary) by another (controlling company).

Determining the degree of competition in an industry is a topical issue in Mexico. In works such as that of COFECE (2017) (Federal Commission of Economic Competition, Spanish: Comisión Federal de Competencia Económica), the effect of groups of AFORES (retirement funds) that make agreements to be benefited is analyzed, for example, in the aspect of decreasing competition for their clients and the commercial expenditure they have made.

The application of the "rock-paper-scissors" model in the analysis of markets, not necessarily the banking market, has antecedents in works such as Hopkins and Seymour (2002), who showed oligopoly as a game of "rock-paper-scissors." In some cases, oligopoly can even be considered collusion (Maskin and Tirole, 1988).

Banks have products that are very similar to each other, for example, the "Gold" credit card market (middle segment). This product can be found in most banks, with a name change, but with the same benefits. This indicates that there may be competition for offering a similar product to the consumer. Therefore, a comparison between banks can be made assuming they offer very similar products.

Table 1 compares the "Gold" cards of the three largest banks[1] in Mexico (BBVA Bancomer, Santander, and Citibanamex).

Table 1
Benefits of credit cards

	Banks		
Benefits	A	B	C
Zero Annuity			☑
Points	☑	☑	☑
Minimum amount	☑	☑	☑
Purchasing in installments		☑	
Mobile App	☑	☑	☑

Source: Gold Cards (according to bank portals), created by the authors using Excel.

As can be seen, they share very similar characteristics. Although the comparison does not cover all contractual aspects of each product, it was made with the information available to the public on their respective websites.

The Mexican credit system

The Mexican financial system has gone through great changes and moments of crisis, one of the best known being the "Tequila crisis," the study of which has been the subject of research, as in Hernández and López (2000), Murillo (2002), and Goméz (2014). In these works, it is concluded that years before the crisis and even after it, with its consequent privatization, banks maintained low Herfindahl Hirschman (HH) indexes (i.e., they had no concentration) in their commercial loan portfolio for the periods between 1991 and 2000 (Murillo, 2002). According to the weighting obtained from The United State Department of Justice (2010), the following concentration levels for the HH index are indicated:

a) Values below 0.01 indicate high competitiveness.
b) Values less than 0.15 indicate no concentration.
c) Values greater than 0.15 and less than 0.25 indicate moderate concentration.
d) Values greater than 0.25 indicate high concentration.

[1] The information was obtained from their websites.

The academic discussion on bank concentration usually revolves around the characteristics of the products and the degrees of concentration. If only this information is considered, it is possible to think that the market behavior corresponds to a cartel. However, works such as COFECE (2017) provide empirical evidence to affirm that a trust exists, given the public evidence of bank associations and the stability of market shares.

The Mexican banking system's loan portfolio comprises the 60 institutions shown in Table 2.

Table 2
Financial Institutions considered in the study

Institution					
1	HSBC	21	Banco Azteca	41	Pagatodo
2	GE Money	22	Autofin	42	Forjadores
3	Ixe	23	Barclays	43	Inmobiliario Mexicano
4	Inbursa	24	Compartamos	44	Dondé Banco
5	Interacciones	25	Banco Ahorro Famsa	45	Bancrea
6	Banca Mifel	26	Multiva	46	Finterra
7	Scotia Bank	27	Actinver	47	ICBC
8	Invex	28	Banco Wal-Mart	48	Shinhan
9	Bansí	29	Intercam Banco	49	Mixuho Bank
10	Afirme	30	BanCoppel	50	Bank of China
11	Accendo Banco	31	ABC Capital	51	Banco S3
12	American Express	32	Biafirme	52	KEB Hana Bank
13	Bank of America	33	Consubanco	53	Banamex Consolidado
14	MUFG Bank	34	Volkswagen Bank	54	BBVA Bancomer Consolidado
15	J. P. Morgan	35	Banco Deuno	55	Santander Consolidado
16	Monex	36	CIBanco	56	Banco del Bajío Consolidado
17	Ve por Más	37	The Bank of New York Mellon	57	Sabadell Consolidado
18	ING	38	Banco Base	58	Banregio Consolidado
19	Deutsche Bank	39	Banco Bicentenario	59	Banorte Consolidado
20	Credit Suisse	40	Bankaool	60	Total Consolidated Multiple Banking

Source: created by the authors with data from the CNVB (2020), referring to Multiple Banking

Figure 1 shows the performing loan portfolio and its monthly growth from December 2007 through November 2019.

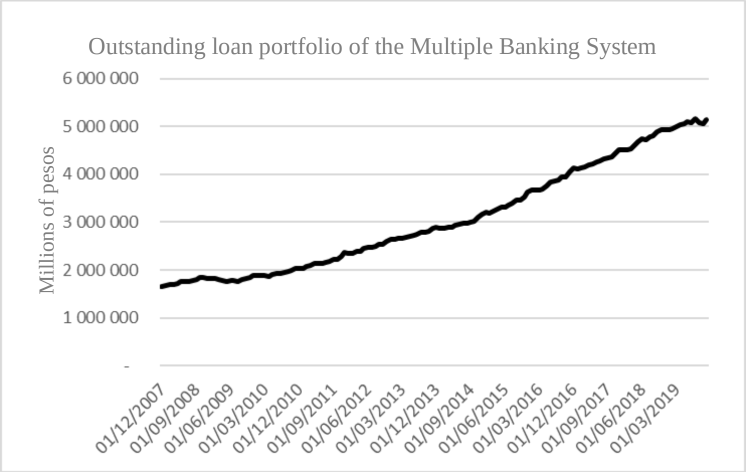

Figure 1. Growth of the total performing portfolio of commercial banks
Source: created by the authors with data from the CNVB (2020), referring to Multiple Banking

The asset concentration of banks according to the HH index, based on data from the National Banking and Securities Commission (CNBV) (Spanish: *Comisión Nacional Bancaria y de Valores*) for the same period, is shown in Figure 2.

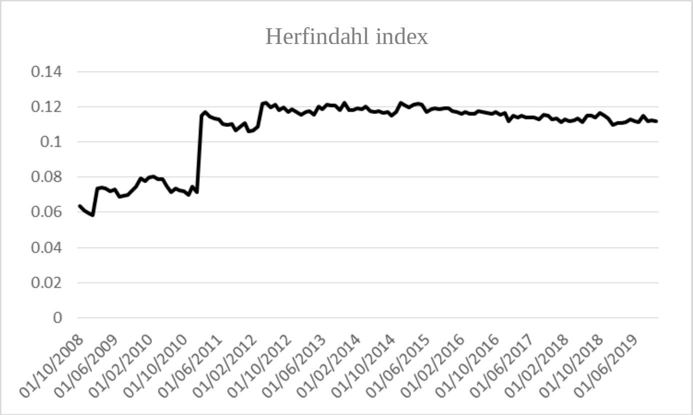

Figure 2. Normalized Herfindahl-Hirschman Index
Source: created by the authors with data from the CNVB (2020), referring to Multiple Banking

It can be seen that in this case the HH index indicates that there is no concentration.

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In the Mexican banking system, there are 12 institutions[2] with more than one hundred thousand cards issued, of which BBVA Bancomer, Banamex, and Santander have a 61% market share in number of credit cards, and 67% if the credit balance granted is considered, Banxico (2021). Therefore, when analyzing the change in strategies among these banks, the results apply to the entire market.

When considering the size of their assets and loan portfolio individually, the three most representative agents of the system—BBVA Bancomer, Santander, and Banamex—will be considered as the system's total in this paper. The market share held by each of them is shown in Figure 3.

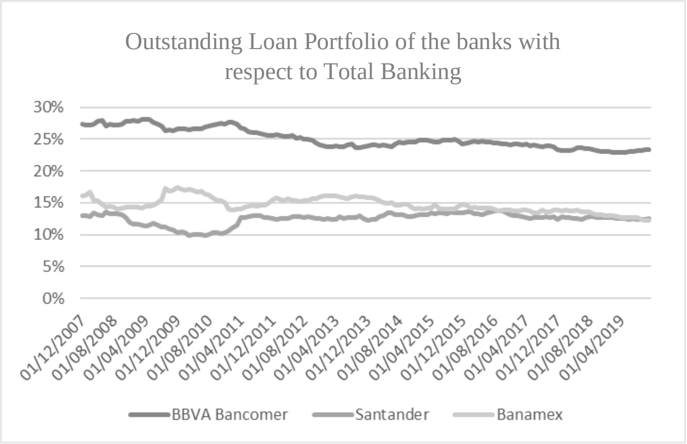

Figure 3. Percentages of Portfolios of the main banks in Mexico
Source: created by the authors with data from the CNVB (2020), referring to Multiple Banking

It can be seen that the behavior of the three is similar since market shares remain partially stable.

Considering the three banks as the total Mexican banking system, Figure 4 is obtained, which represents the values for the rock-paper-scissors model.

[2] HSBC, Banregio, Santander, Banamex, American Express, Inbursa, Invex, BBVA Bancomer, Banorte, Scotiabank, Banco Famsa, and BanCoppel

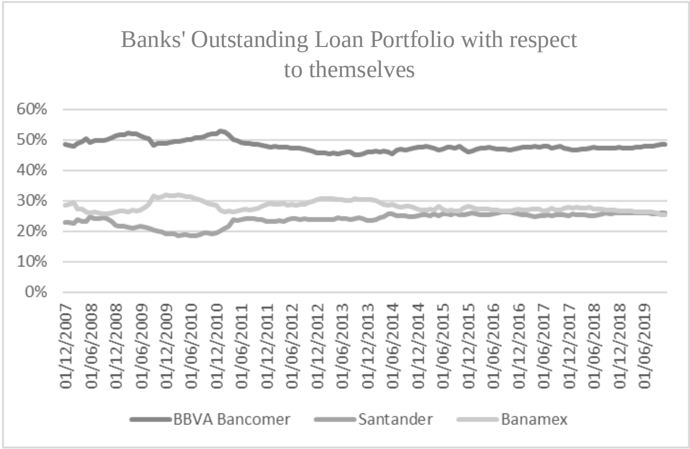

Figure 4. Percentages of portfolios of BBVA Bancomer, Santander, and Banamex banks
Source: created by the authors with data from the CNVB (2020), referring to Multiple Banking

The proposed model links the market share of each of these banks to the strategy of the remaining two, allowing it to mutate (change) in the face of another player's success. It also shows that, at least in credit concentrations, the market is competitive since the response time is around three months, the time in which credit ratings are reevaluated. This evidence suggests a form of monopolistic competition.

Methodology

The study is performed through stochastic differential equation systems for a dynamic evolutionary game of the Nowak (2006) type, or from the non-cooperative evolutionary strategies point of view of Zhou (2016), applied to the "rock-paper-scissors" game.

The main contribution of the present work is the inclusion of a Brownian motion in the equation that models the payment function of each type of bank, and that of a mutation factor to model the effect of asymmetric information, which makes it possible to measure the degree of industrial competition through the mutation factor.

The outcome of the "rock-paper-scissors" game can be expressed by the payoff ratio of each agent in the face of each strategy of its adversaries, as shown in Table 3.

Table 3
The rock-paper-scissors problem as a function

Possible strategies		Rock P_1	Paper P_2	Scissors P_3
Rock	P_1	0	- ϵ	1
Paper	P_2	1	0	- ϵ
Scissors	P_3	- ϵ	1	0

Source: created by the authors with Excel

From Table 3, it follows that P_i,i=1,2,3, which corresponds to the densities of each player. s(t) is introduced with elements such as s_i,i∈(P_1,P_2,P_3), whose replication equation is given by (s_i) ̇=[π_i-π ̅].

In addition to the above, there is the mutation factor, which allows each player to change strategy, as shown in (1).

$$P_1 \overset{\mu}{\rightarrow} \begin{cases} P_2 \\ P_3 \end{cases}, \; P_2 \overset{\mu}{\rightarrow} \begin{cases} P_1 \\ P_3 \end{cases}, \; P_3 \overset{\mu}{\rightarrow} \begin{cases} P_1 \\ P_2 \end{cases} \tag{1}$$

According to Mobilia (2010), (1) can also be described by the System of Equations (2), which indicates the dynamics of the system.

$$\begin{aligned} \dot{P}_1 &= P_1\big(P_3 - \epsilon P_2 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_1) \\ \dot{P}_2 &= P_2\big(P_1 - \epsilon P_3 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_2) \\ \dot{P}_3 &= P_3\big(P_2 - \epsilon P_1 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_3) \end{aligned} \tag{2}$$

where:

P_i: It is the competitor $i = 1, 2, 3$. In this paper, they represent banks.

ϵ: Represents the competitive advantage

μ: Is the probability of strategy change, which can be a function of time or a function of adaptation of an evolutionary game strategy.

This system can be viewed as a dynamic game (Nowak, 2006), or from the point of view of non-cooperative evolutionary strategies (Zhou, 2016). By using:

$P_1(t) + P_2(t) + P_3(t) = 1$ from (1) as $P_3(t) = 1 - P_1(t) + P_2(t)$,

then the market share of player three is presented as a function of the other two players.

By incorporating a stochastic process $dW_i(t)$ in the System (2), through a standard Brownian motion, it follows that $0 < s < t$, so that the increments in $W_{t_i} - W_{t_{i-1}}$ are $N(0, t - s)$ (Venegas Martinez, 2008).

If the population is considered to be of size $N < \infty$, the sample average approximates the rate of the Equation and System (2) is transformed into System (3):

$$\dot{P}_1 = P_1\big(P_3 - \epsilon P_2 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_1) + dW_1(t)$$

$$\dot{P}_2 = P_2\big(P_1 - \epsilon P_3 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_2) + dW_2(t)$$

$$\dot{P}_3 = P_3\big(P_2 - \epsilon P_1 - (1-\epsilon)(P_1P_2 + P_2P_3 + P_1P_3)\big) + \mu(1 - 3P_3) + dW_3(t) \tag{3}$$

By (2) and in the absence of mutation, $\mu = 0$, an equilibrium (or fixed point) is obtained:

$$s^* = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

This fixed point s^* indicates the market share. It is also noted that including Brownian motion within each player's strategy change implies introducing adaptability to each player.

One of the major differences between classical game theory, where the best strategy implies a mixed Nash equilibrium, is that this approach allows the possibility of adaptation of each player to be incorporated at least partially. It is reasonable to assume that, during a changing game environment, the payoff functions and the system's equilibria are also changing, at least in the short term. This adaptive role is taken by the mutation factor μ.

To emphasize the importance of adaptation in the game, cases without mutation ($\mu = 0$) and with mutation ($\mu \neq 0$) are analyzed in this paper.

The competitive advantage parameter, ϵ, can take the following values:

- $\epsilon < 1$: Indicates an attractor system
- $\epsilon = 1$: Is a stable point
- $\epsilon > 1$: Leads to an unstable system, and the trajectory leaves the phase portrait

It is important to emphasize that including a mutation or strategy change factor gives stability to the system.

Case study

To analyze the speed and type of adaptation of Mexican banks' lending practices in the face of changes in their competitors' strategies, a non-zero value for the mutation parameter μ is considered and calculated with a stopping time in the established months, considering $\sum_{i}^{3}\left|s^* - \widehat{P}_i\right| \leq 0.05$, where s^* are the interior fixed points mentioned in the previous section.

Approximations to the solutions s^* of the differential equations are determined using the 4th-order Runge Kutta numerical method (Chapra et al., 2007), as in the case of Equations (1). Then, three-time intervals for the mutation parameter μ are considered:

- 1 month for a product that is easy to imitate, such as changing a promotion on a credit card
- 6 months for a more elaborate product that requires a testing process (it could be a new internal software or a mobile application) since large corporations have qualified personnel or outsourcing that guarantee to deliver a copy of a given product
- 3 months for the mid-point between a simple and an elaborate product

In the solutions, mutation parameters μ will be expected to be very high (close to 1) if credit institutions adapt quickly, and very small (close to zero) if there are strong entry barriers, making adaptation difficult. The result can be an indirect number of the degree of competition in an oligopolistic system.

The resulting mutation parameter μ in the three study periods is shown in Table 4.

Table 4
Estimates for μ

Month	Value found
1	0.593574345
3	0.223951997
6	0.126525566

Source: created by the authors with Excel

The value of the mutation parameter μ indicates the sum of the differences of a theoretical stable point s^* against the agent's market share P_i. The solutions show that institutions adapt quickly when the product is easy to imitate, whereas as the degree of complexity increases, the adaptation time increases.

Assuming the value of comparative advantage ϵ, and following the methodology proposed by Kerr et al. (2002) for the calculation of the fitness of one player with respect to another, the following is obtained:

$$w(x, y) = \frac{\ln(x_F/x_0)}{\ln(y_F/y_0)}$$

where w is the competition between x players relative to y and x_0, y_0 are the initial densities, and x_F, y_F are the densities after a period of activity. This calculation leads to a value of the competitive advantage of one bank relative to another of $\epsilon = 1.54$.

This paper aims to analyze the institutions' speed and type of adaptation; therefore, the mutation factor used in the simulations of the proposed model is $\mu = 0.22$. This result is compatible with the

Mexican banking environment since, empirically, this type of product has an adaptation period of between 3 and 6 months.

The results of the solution simulations of the system—of differential equations—are shown below, considering the classical model with no stochastic mutation factor and the solutions with its inclusion. Figures 5 to 8 show the system simulations.

Figure shows the simulation of the system solutions in the classical case, with $\epsilon = 1.54$, and zero probability of banks changing strategy, with no stochastic movement ($\mu = 0$); i.e., the classical case.

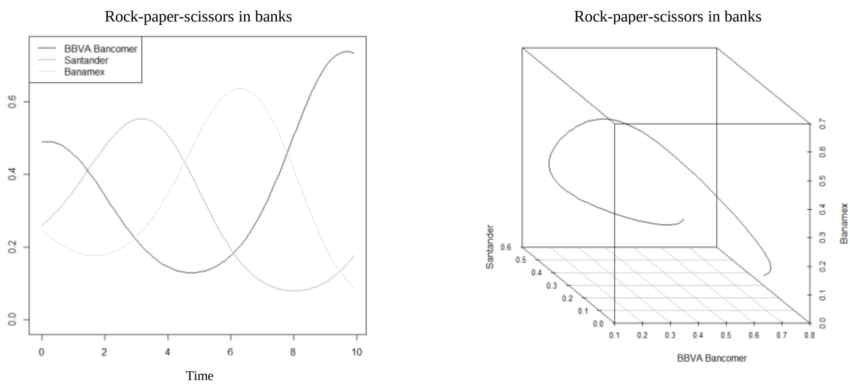

Figure 5. Simulation benches in the rock-paper-scissors model $\epsilon = 1.54, \mu = 0$
Source: created by the authors with R-project and the Sim.DiffProc package

As ϵ=1.54, the interior point is unstable, and the phase portrait forms a heteroclinic connection (periodic or chaotic solutions) if the mutation rate remains at zero (μ=0); that is, if the banks do not innovate. This generates a state of instability in the system.

Considering that every time a bank innovates a loan product and the other banks try to modify their plans within 3 to 6 months to avoid being left out of the market, the solution simulations shown in Figure 6 are obtained.

http://dx.doi.org/10.22201/fca.24488410e.2022.2958

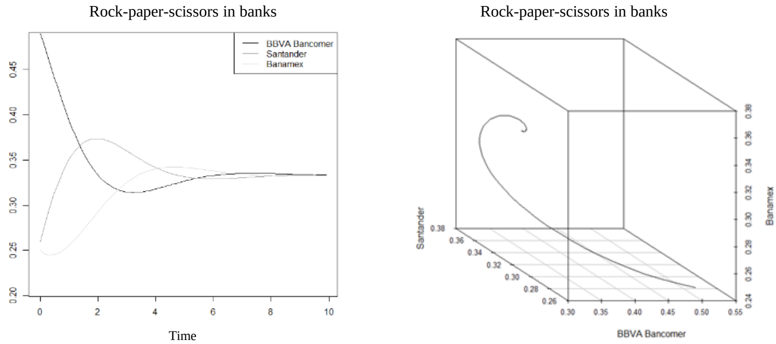

Figure 6. Simulation of banks in the rock-paper-scissors model ϵ=1.54, μ=0.22
Source: created by the authors with R-project and the Sim.DiffProc package

The model's outcome changes radically when strategy mutation is allowed (which is intuitively plausible in an environment with some degree of competition). A process of competitive adaptation is generated, and the phase portrait forms stable equilibrium trajectories.

The stochastic mutation parameter is not considered in the previous models but is included in the system results shown in Figure 7, with $\epsilon = 1.54$, and assuming no innovation by the competition $\mu = 0$, with stochastic motion.

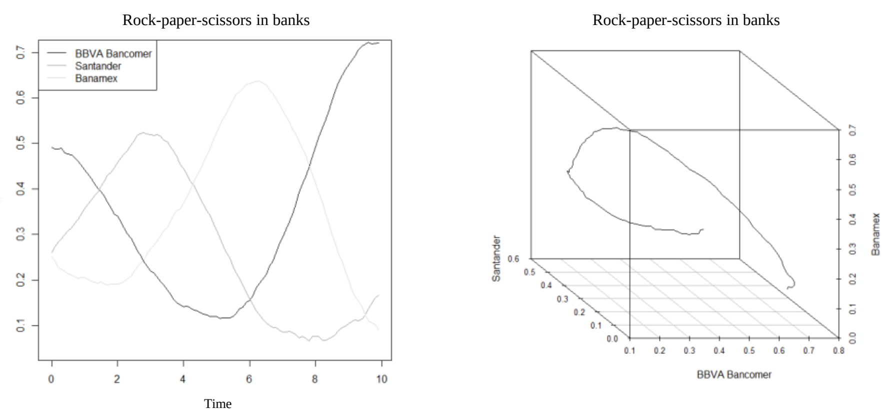

Figure 7. Simulation of banks in the rock-paper-scissors model $\epsilon = 1.54, \mu = 0, dW_t$
Source: created by the authors with R-project and the Sim.DiffProc package

The results show that if there is no innovation in competition, instability is generated in the system, and the Mexican credit system may collapse due to market concentration.

Finally, Figure 8 shows the results of the system simulations when a stochastic motion is considered in the competition adaptation process. A Brownian motion without jumps was included in the system, which assumes that the unseen variables follow a normal distribution μ=0.22,dW_t and considering ϵ=1.54.

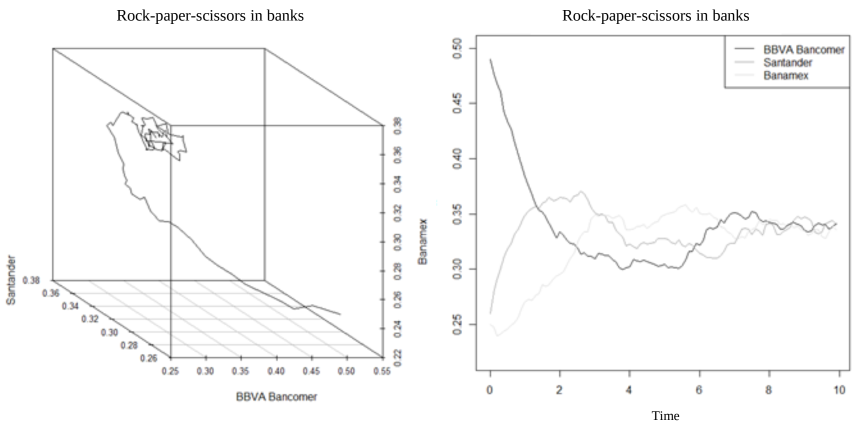

Figure 8. Simulation in the rock-paper-scissors model $\epsilon = 1.54, \mu = 0.22, dW_t$
Source: created by the authors with R-project and the Sim.DiffProc package

The result shows that a process of adaptation to competition is generated through stable equilibrium trajectories. It can be observed that, although there are movements in the short term, it moves towards a stable point over time.

The above figures show the stabilizing effect of the mutation parameter on the solution system and its ability to approximate the barriers to innovation through imitation.

Conclusions

In an environment of imperfect competition with incentives for companies to collude and whose effects are costly for consumers, it is important to know the degree of competition in an industry. The way to increase market share is by increasing risk or through technological changes. When mutation rate is incorporated, it is postulated that there is a rate of change in a company's strategy with respect to its rivals; the greater the competition, the easier it is for products to be replicated.

This paper analyzed the speed and type of adaptation of Mexican banks in the granting of credit through differential equation systems for a dynamic evolutionary game of the Nowak (2006) type, or from the point of view of non-cooperative evolutionary strategies of Zhou (2016), applied to the "rock-paper-scissors" game.

Together with the above, a tool is developed to measure the degree of competition through the mutation parameter with the market strategies; with this calibration, market competition data is obtained. This parameter moves until it is equal to the observed times, which makes it possible to measure the degree of competition through the degree of product replication.

The results show that for solution trajectories to be stable, competitors must adapt strategies when one of them innovates with a product with a medium adaptation time of 3 to 6 months. The non-zero mutation parameter becomes a hidden variable that measures the degree of competition and governs the time variable of the adaptation time.

The main contribution of the present work is the inclusion of a Brownian motion to the equation that models the payment function of each type of bank, and that of a mutation factor to model the effect of asymmetric information, which allows the degree of industrial competition to be measured through the mutation factor.

One of the main limitations of this work is that, although there are thresholds of stability in the system of adaptation to high competition in the Mexican credit market, the model does not consider cases in which the degree of innovation is small or large. This task remains pending for future work.

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