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Estimation of reference prices for international trade using conics

Estimación de precios de referencia para comercio internacional mediante cónicas

Sergio Lagunas Puls*[1], Juan Bautista Boggio Vázquez

Universidad del Caribe, México

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Abstract

It is proposed to use conics as a reference price method, contributing to the verification of the value of goods at customs, according to the international classification of the Harmonized Commodity Description and Coding System (HCDS). After the introduction, the SADC and elements related to the scope operations in this work are described. The following section exposes substantive elements contained in previous studies regarding the importance of the customs value, giving way to the proposed methodology, based on the geometry of hyperbolas explained through implicit derivation; subsequently, the data and modeling are described, exemplifying with coffee exports from Brazil, showing how unit price estimates could be made, assuming real time, with respect to reference prices in other periods. In conclusions, it highlights that this proposal could mean a new alternative for estimating transfer prices, and it would also contribute to optimize resources by discarding from review or audit those operations that do not suggest undervaluation or overvaluation.

JEL Code: F17, F14, C02
Keywords: international trade; harmonized system; auditing; customs

*Corresponding author.
E-mail address: slagunas@ucaribe.edu.mx (S. Lagunas Puls).
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Resumen

Se propone utilizar cónicas como método de precios de referencia, contribuyendo con la verificación del valor de mercancías en la aduana, de acuerdo con la clasificación internacional del Sistema Armonizado de Designación y Codificación de Mercancías (SADC). Después de la introducción, se describe el SADC y elementos relacionados con las operaciones de alcance en este trabajo. El siguiente apartado expone elementos sustantivos contenidos en estudios precedentes respecto a la importancia del valor en aduana, dando paso a la metodología propuesta, basada en la geometría de hipérbolas explicada mediante derivación implícita; posteriormente se describen los datos y el modelado, ejemplificando con exportaciones de café desde Brasil, mostrando cómo podrían realizarse estimaciones de precios unitarios, suponiendo tiempo real, con respecto a precios de referencia en otros períodos. Destaca en conclusiones que esta propuesta podría significar una nueva alternativa para estimar precios de transferencia, además que contribuiría a optimizar recursos descartando de revisión o auditoría a las operaciones que no sugieran subvaluación o sobrevaluación.

Código JEL: F17, F14, C02
Palabras clave: comercio internacional; sistema armonizado; fiscalización; aduanas

Introduction

Fiscal discipline, a fundamental element in Mexico's economic plans (Presidency of the Republic, 2019a), requires permanent monitoring. For this reason, proposals aimed at safeguarding public resources and supporting the business sector by evaluating prices in international trade are of great interest. Hence, it would be possible that, taking advantage of international trade statistics generated day by day in the systematized reports for the dispatch of goods, authorities and individuals could consult prices by tariff code, whether observed at a specific time or in real time, or obtain future expectations according to trajectories.

Therefore, this paper aims to use conics as a transfer pricing method (Griffin, Brekel & Coronado, 2018; OECD, 2017), contributing to verifying the value of goods at customs (Gobierno Federal, 2019) as a measure prior to tax and customs checks (Mancilla-Rendón, 2010; Torres & Chávez, 2015). The hypothesis is that through conics—curves that have been used for the explanation of supply and demand theory (Ruiz, 1997; Cómbita Mora, 2012)—a visual representation of trade and price expectations for any tariff code can be obtained, providing stakeholders with an alternative pricing method.

Conics offer representations well suited to international trade (García-Pérez et al., 2016). Hyperbolas reflect in two quadrants—one for the exporter and the other for the importer—the position of a phenomenon (García-Fernández et al., 2014), in this case, the quantity of goods and the agreed price. Parabolas, which can be presented with horizontal or vertical openings, offer different scenarios with greater or lesser volatility for trade operations and expectations (Lagunas & Cervantes, 2021).

This way, the authority could focus on situations that suggest the presence of distorted prices, optimizing public resources and the resources of individuals, and avoiding management and proof costs (Congreso de la Unión, 2006).

According to Article 180 of the Income Tax Law (LISR) (Spanish: Ley del Impuesto Sobre la Renta), there are 6 methods to establish transfer prices between related parties in Mexico for 2021 (Congreso de la Unión, 2016). For its part, Article 70 of the Customs Law (LA) (Spanish: Ley Aduanera) establishes three methods for valuing goods when there is a relationship between the importer and the exporter (Congreso de la Unión, 2018). Therefore, the proposal in this article is complementary since, in addition to being a methodology that has not been applied, it could be used in all types of commercial transactions and not only in those involving related parties.

The paper begins with the background of the operations whose international prices could be evaluated, mentioning the essential concepts and outline of the commercial situation. Next, the regulation and importance of Mexico's economic and commercial dynamics are discussed, followed by a description of the methodology and an example of the proposal.

It is concluded that a greater number of international operations could be verified through the same methodology, with scope for the entire Harmonized Commodity Description and Coding System (HCDS) and for operations between related parties according to the interest of each country in particular.

International trade of goods

It is essential in the international trade of goods to establish the transaction value or customs value (Troncoso Consultores en Comercio Exterior y Aduanas, 2019), which is the basis for paying taxes. This value is based on the price paid, plus, in some cases, items known as incremental expenses. In this regard, the Mexican Customs Law provides that if the value is not clearly identified, alternative methods must be used until an acceptable value is obtained, applying only one of the methods but ensuring that it is in the following order: transaction of identical goods, transaction of similar goods, unit sale price, and reconstructed price of the goods (Congreso de la Unión, 2018)[1].

Once the customs value has been determined, it is necessary to consider the provisions of the General Import Tax (IGI) (Spanish: Impuesto General de Importación) for the payment of duties, which may be expressed as a percentage rate on the customs value, a fixed quota established for a certain quantity or per unit of merchandise, or a combination of both (mixed). In addition, international merchandise

[1]In addition to the methods provided for in the Mexican Customs Law, at the international level there is also a method called "Last Resort Method" (World Trade Organization, 2019).

transactions may be subject to non-tariff requirements, such as phytosanitary requirements, compensatory quotas and customs processing fees, depending on the type of merchandise, origin, and destination.

In 1988, the Convention to implement the Harmonized Commodity Description and Coding System (HCDS), which structures the international codification of trade using six digits (World Custom Organization, 2017) called tariff subheadings, under which all goods are classified for better identification, entered into force to meet the requirements for each operation. It is important to note that, in addition to the international use and structure, each country may add digits to the six internationally accepted digits if it considers that greater specificity is necessary to improve controls and analysis. For example, Mexico adds two, forming a Tariff Fraction with an eight-digit structure[2].

More than 98% of world trade has been codified through the HCDS (World Custom Organization, 2012), and it has facilitated the identification of goods, making it a crucial element for the conclusion of international treaties or agreements (Asociación Latinoamericana de Integración, 2019). It also became a tool to minimize errors in collecting contributions or to contest or exempt certain incorrectly determined requirements. The codification of the HCDS has created important databases, grouping the commercial dynamics of transactions as country/country, country/countries, country/trading partners, and even at regional and global levels. In the case of Mexico, the statistics grouped in HCDS codes and tariff fractions can be consulted in the Tariff Information System via Internet (Secretaría de Economía, 2019), but also on international sites such as the International Trade Centre (ITC), constituted as a coordination point of the United Nations (UN) for technical assistance in international trade (International Trade Centre, 2019).

According to the ITC, worldwide merchandise imports reached 19 689 785 570 000 dollars in 2018, while in the case of Mexico, they totaled 464 268 470 000 dollars, which mainly came from the United States of America, China, Japan, Germany, and Korea. Mexico's top five international merchandise suppliers have consolidated over time, as indicated by consultations of the ITC from 2010 to 2017 (International Trade Centre, 2019). Nonetheless, imports from China have constantly grown, with marked stability in imports from Germany, Japan, and Korea, and major variations in goods shipped from the United States of America.

It is important to understand that imports are responsible for the public resources that international trade generates for Mexico. Imports enable the collection of tariffs, Value Added Tax (VAT), the Special Tax on Production and Services (IEPS) (Spanish: Impuesto Especial sobre Producción y Servicios), and even compensatory duties. For 2017, the total collection in Mexican customs reached the amount of 845 294 070 727 Mexican pesos, of which 74.33% corresponded to VAT, 18.84% to IEPS,

[2]The HCDS at the international level employs a six-digit coding system. From left to right, the first two correspond to the chapter, four correspond to the heading, and six to the tariff subheadings (World Custom Organization, 2012).

6.27% to IGI, and the rest to customs processing fees (DTA) (Spanish: Derechos de Trámite Aduanero), taxes on new cars (ISAN) (Spanish: Impuestos sobre Automóviles Nuevos) and other items (Servicio de Administración Tributaria, 2019a).

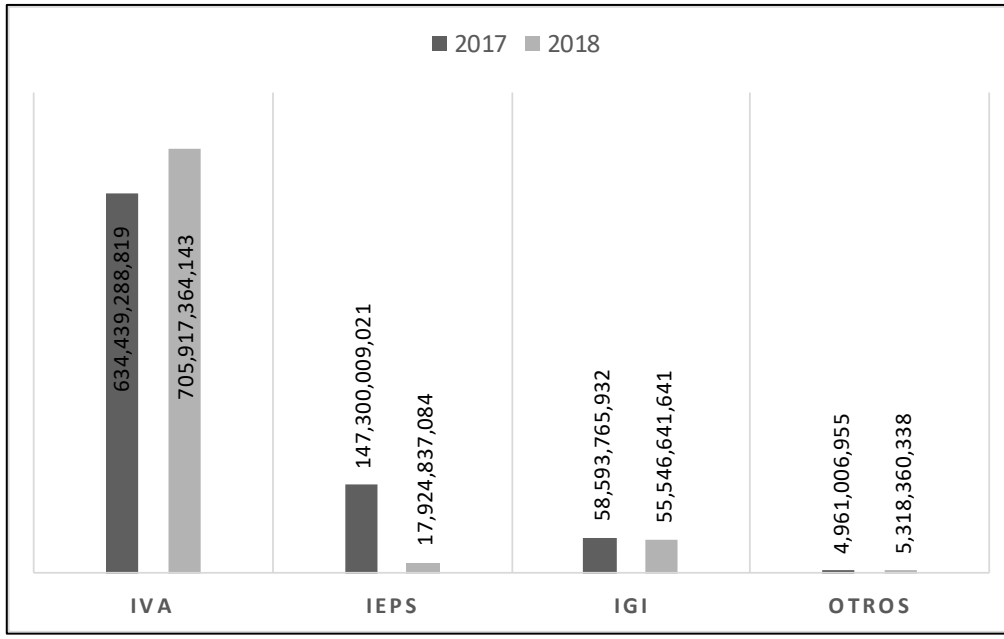

Figure 1. Total revenue from customs operations in Mexico
Source: created by the authors, data from the Tax Administration Service

According to Figure 1, total customs revenue increased in 2018. Nevertheless, its composition did not have many changes: it totaled 949 707 203 206 Mexican pesos, of which 75.06% was VAT, 17.43% was IEPS, 6.93% was IGI, and the rest was due to other charges (Servicio de Administración Tributaria, 2019btroncos).

Although in some countries, such as Mexico, exports are exempt from paying tariffs and are subject to 0% Value Added Tax (VAT) for reasons of efficiency, competitiveness, and strengthening domestic production, it is not only necessary but urgent to analyze them in order to contribute to market diversification (Presidency of the Republic, 2019b; Ministry of Foreign Affairs, 2019).

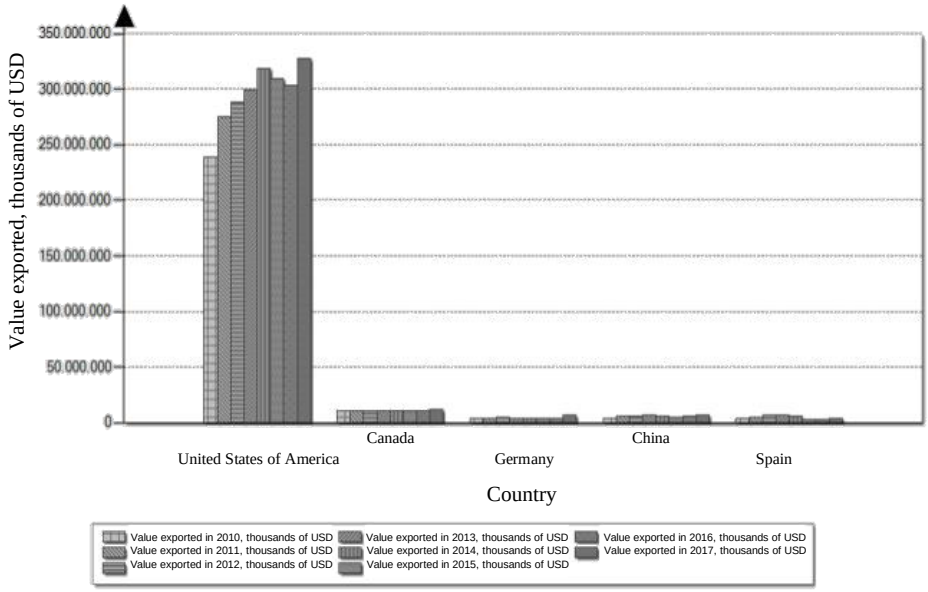

Figure 2. Supplier markets for a product imported into Mexico
All products
Source: International Trade Centre

Figure 2 shows the great inequity in commercial presence by region, where the United States of America is not only the main destination for Mexican exports but also the main supplier, representing a consolidated opportunity while at the same time a serious risk due to the level of dependence that this situation generates.

Customs valuation and the prevention of undervaluation

It is necessary to emphasize that the substantive element for ad-valorem operations is the acquisition price to achieve adequate collection. Nonetheless, when the value is considered distorted, 4 alternative methods are applied in Mexico[3] (Congreso de la Unión, 2018); moreover, according to the Customs Law, when the impossibility to correctly determine the value is argued, the interested parties may seek a consultation with the General Legal Administration (Servicio de Administración Tributaria, 2020).

On the other hand, the World Customs Organization (WCO) complements Mexico's methods by adding the so-called "last resort" method, indicating that it "should be based on alternative values and

[3]The methods referred to in the Mexican Customs Law are: transaction of identical merchandise, transaction of similar merchandise, unit sales price, and reconstructed price of the merchandise.

methods, but with reasonable flexibility in its application," i.e., a combination of two or more methods with a reasoned explanation (World Trade Organization, 2020).

The WCO publishes a report with the main global analyses annually. Among these are the detection of illicit drugs, weapons, pharmaceutical items, or chemical compounds, but with a common element that, according to the 2018 report, is the safeguarding of the payment of taxes through the correct commercial valuation (World Customs Organization, 2018). This report makes it clear that it is imperative to monitor that prices correspond to market values in international trade, either in the country of origin or for trade in the destination country, and also that they can be compared with references in international statistics (World Custom Organization, 2019).

As can be seen, the formation of value is critical. Therefore, the academic community has been addressing the issue with different approaches, converging on the importance of the declared or determined value. Thus, the author González-Bianchi (2019) compares situations in which anomalies can be identified, calling them errors that are pointed out and sanctioned by administrative, tax, and criminal laws. Nonetheless, given the commercial nature, there is a possibility of carrying out corrections that exempt from responsibility but also from full compliance with the obligations derived from the commercial operation.

Such is the importance of a correct customs valuation that some countries and regions have proposed not only valuation methods but a total structural reform for trade that provides authorities with technological powers and tools to understand behavior patterns and improve tax collection. Such tools could verify prices and improve recognition procedures (Montagnat-Rentier, 2019).

Other studies propose applying technological tools to prevent undervaluation. In this respect, Vanhoeyveld, Martens, and Peeters (2019) analyzed more than nine million transaction records to identify behavior patterns through mechanisms applied in generic fraud. Nonetheless, they concluded that the international area is rather complex and that traditional algorithms needed to be adjusted to patterns with global characteristics; therefore, they proposed a vector-based methodology using data compiled by the Harmonized Commodity Description and Coding System. For this reason, this study is considered a relevant frame of reference for the present study since it recognizes the same intention, although it is not based on conics.

Another study related to the topic of this paper deals with a methodology that identifies products that may present distorted prices and then searches for similar operations that may relate companies that habitually carry out this type of trade. In this way, the authors propose to discriminate among the cases and focus on detecting the relation between commercial parties. Unlike the study by Vanhoeyveld, Martens, and Peeters (2019) that considers analysis by tariff grouping, this one focuses on sectors and

related parties (Choi, 2019), a situation to which the conical methodology proposed in this article could be adapted.

For some authors, verification in Latin America has concentrated on particular operations, applying methods that compare or reconstruct costs, prices, or profits (Ilha, Mandía, & Pastor, 2011). Some have a particular focus on participation in production chains, seeking to predict and analyze scenarios that help decision-making; others have an emphasis on the impact of public policies, performance by level, and price determination (Masaro, 2019). Therefore, the conical methodology described in this article could also be proposed, establishing curves in production chain links.

As a result, the opportunity arises to present prior assessments, providing commercially customary intervals. This gives the authorities and interested parties an opinion before verifying or auditing individuals, allowing them to permanently evaluate the commercial dynamics and direct tax reviews outside the acceptable intervals in those cases. Additionally, it would contribute to strengthening national production, avoiding unnecessary expenses while strictly monitoring public resources, a situation of special interest for the President of Mexico (Presidency of the Republic, 2019a).

In addition to the substantive importance of value in customs, which is the topic of this article, it is aligned with the recognition of international organizations, such as the Latin American Integration Association (ALADI) (Spanish: Asociación Latinoamericana de Integración), which recognize the need to harmonize mechanisms and procedures for evaluation and control in international trade. This is important since, even in 2018, Colombia has not yet harmonized its mutual evaluation and recognition procedures. Uruguay has not achieved the sharing of single origin formats or the still partial proposal to allow invoicing for imports to be carried out by third parties (Asociación Latinoamericana de Integración, 2019).

Methodology

The models are based on conic sections of hyperbolas and parabolas (González-Mazuelo & Patiño-Jaramillo, 2007; Cantón, Fernández-Jambrina, & María, 2011; Weiss, 2017), specifically those that express the essence of a sale of goods,—those who export and obtain an income—that is, from the zero to the right of the x-axis, expressed in monetary units. Conversely, for those who purchase goods (import), the consideration is negative for the payment they make, i.e., from the zero to the left on the x-axis (thousands of dollars). For both cases, the quantities of goods are the same and are represented on the y-axis in the units in which the tariff subheading is expressed, according to the HCDS.

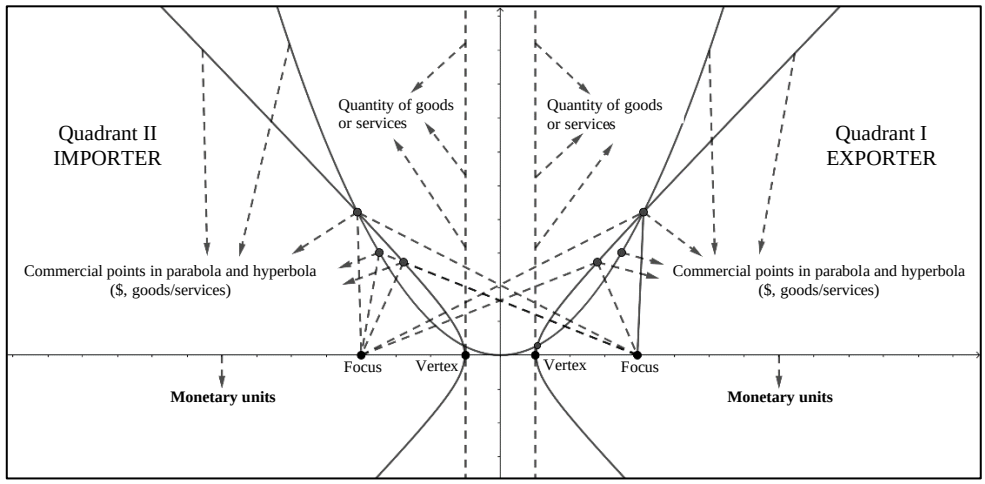

Figure 3. Fundamental elements for conic models
Source: created by the authors using GeoGebra

Considering that the origin of a commercial operation requires the existence of goods (positive value), only the conics that pass through quadrants I and II, as shown in Figure 3, are considered, i.e., those with correspondence for two economic agents that essentially participate in any international operation, one as an exporter that obtains income (quadrant I), and the counterparty that acquires and pays for the imported goods (quadrant II).

Hyperbolas offer more stable trajectories (Santiago, Alcalá, Ortiz, & Lozano, 2010). They are usually less pronounced, which should be interpreted as a lower elasticity of demand (a change in prices affects, but only weakly, the quantities of goods demanded), predicting that trade does not fall beyond the vertex due to habitual trade.

Parabolas offer estimates for goods or markets with greater volatility (Fanelli & Jiménez, 2009), i.e., with greater elasticity (if prices increase, this significantly reduces the number of goods sold or demanded), as shown in Figure 3.

The following is a description of how to obtain the elements for parabolas and hyperbolas (González-Mazuelo & Patiño-Jaramillo, 2007) suggested for the valuation of international trade operations:

The following form expresses the conics in a general way:

$$\delta x^2 + \beta xy + \varsigma y^2 + \phi x + \psi y + \xi = 0 \quad (1)$$

All parabolas have a vertical focal axis at the vertex (0,0), therefore:

$$\varsigma = 0, \delta \neq 0 \tag{2}$$

To locate the points on the graphs, differential calculus is used where the derivative is either zero or undefined

$$\frac{dy}{dx} = 0, \frac{dy}{dx} = nd \tag{3}$$

given the implicit relation x, y, and the derivation of Equation (10), yielding

$$\frac{dy}{dx} = \frac{-2\delta x - \phi}{2\varsigma y + \psi} \tag{4}$$

The vertices of the conics are located where the following is true

$$\frac{dy}{dx} = 0, si - 2\delta x - \phi = 0 \tag{5}$$

$$\frac{dy}{dx} = nd, si\ 2\varsigma y + \psi = 0 \tag{6}$$

The parabola models are defined as follows

$$\frac{dy}{dx} = \frac{-2\delta x - \phi}{\psi} \tag{7}$$

Tangent lines for parabolas, with vertical focal axis

$$x = h = -\frac{\phi}{2\delta} \tag{8}$$

To locate the value of y, also referred to as point k, (8) is substituted into Equation (1)

$$\delta\left(-\frac{\phi}{2\delta}\right)^2 + \phi\left(-\frac{\phi}{2\delta}\right) + \psi y + \xi = 0 \tag{9}$$

$$-\frac{\phi^2}{4\delta} + \psi y + f = 0 \tag{10}$$

$$y = k = \frac{\phi^2 + 4\delta\xi}{4\delta\psi} \tag{11}$$

Therefore, the vertex will be formed as follows

$$(h, k) = \left(-\frac{\phi}{2\delta}, \frac{\phi^2 - 4\delta\xi}{4\delta\psi}\right) \tag{12}$$

The focus position will be located as shown below

$$p = -\frac{\psi}{4\delta} \tag{13}$$

The coordinates for the focus consist of the following coordinates

$$(h, k + p) = \left(-\frac{\phi}{2\delta}, \frac{\phi^2 - 4\delta\xi - \psi^2}{4\delta\psi}\right) \tag{14}$$

Equation of the directrix

$$y = k - p = \frac{\phi^2 - 4\delta\xi + \psi^2}{4\delta\psi} \tag{15}$$

Equation for the focal axis

$$x = h = -\frac{\phi}{2\delta} \tag{16}$$

The focal width is determined as follows

$$|4p| = \left|-\frac{\psi}{\delta}\right| \tag{17}$$

Concerning the hyperbolas, from Equation (1) and the form for the Derivative (4), the variable y is removed from the latter, which is as follows

$$y = -\frac{\psi}{2\varsigma} \tag{18}$$

Equation (18) is substituted into Equation (1)

$$\delta x^2 + \varsigma\left(-\frac{\psi}{2\varsigma}\right)^2 + \phi x + \psi\left(-\frac{\psi}{2\varsigma}\right) + \xi = 0 \tag{19}$$

Therefore,

$$\delta x^2 + \phi x - \left(\frac{\psi^2 - 4\varsigma\xi}{4c}\right) = 0 \tag{20}$$

$$x = \frac{-\phi \pm \sqrt{\frac{\delta\psi^2 - 4\delta\varsigma\xi + \varsigma\phi^2}{\varsigma}}}{2\delta} \tag{21}$$

Given the type of hyperbola used for the models, it is a requirement that the following characteristic be satisfied

$$\frac{\delta\psi^2 - 4\delta\varsigma\xi + \varsigma\phi^2}{\varsigma} > 0 \tag{22}$$

The values for vertices are determined as follows

$$x_1 = \frac{-\phi + \sqrt{\delta\psi^2 - 4\delta\varsigma\xi + \varsigma\phi^2}}{2\delta} \tag{23}$$

$$x_2 = \frac{-\phi - \sqrt{\delta\psi^2 - 4\delta\varsigma\xi + \varsigma\phi^2}}{2\delta} \tag{24}$$

Consequently, the coordinates for the vertices will be

$$\left(\frac{-\phi \pm \sqrt{\frac{\delta\psi^2 - 4\delta\varsigma\xi + c\phi^2}{c}}}{2\delta}, -\frac{\psi}{2\varsigma}\right) \tag{25}$$

Midpoint between vertices:

$$x_c = -\frac{\phi}{2\delta} \tag{26}$$

Consequently, the coordinates for locating the center of the hyperbola are determined based on the forms (18) and (26), i.e.,

$$C = \left(-\frac{\phi}{2\delta}, -\frac{\psi}{2\varsigma}\right) \tag{27}$$

The distance between C and each of the vertices is obtained as follows

$$a = \left| \frac{\sqrt{\frac{\delta\psi^2 - 4\delta\xi + \varsigma\phi^2}{\varsigma}}}{2\delta} \right| \tag{28}$$

The distance between C and each of the points on the conjugate or imaginary axis (ordinates) is determined as follows

$$b = \left| \frac{\sqrt{\psi^2 - 4\varsigma\left[\delta\left(h \pm \delta\sqrt{2}\right)^2 + \phi\left(h \pm \delta\sqrt{2}\right) + \xi\right]}}{2\varsigma} \right| \tag{29}$$

Therefore, the distance between C and each of the focuses is determined as follows

$$c = \sqrt{\left| \frac{\sqrt{\frac{\delta\psi^2 - 4\delta\xi + \varsigma\phi^2}{\varsigma}}}{2\delta} \right|^2 - \left| \frac{\sqrt{\psi^2 - 4\varsigma\left[\delta\left(h \pm \delta\sqrt{2}\right)^2 + \phi\left(h \pm \delta\sqrt{2}\right) + \xi\right]}}{2\varsigma} \right|^2} \tag{30}$$

Modeling for reference prices

The following is the model structured in GeoGebra to obtain reference prices from the total worldwide export of Brazilian coffee (Vacca, Acosta, & Rodriguez, 2011; Andrade & Petri, 2008).

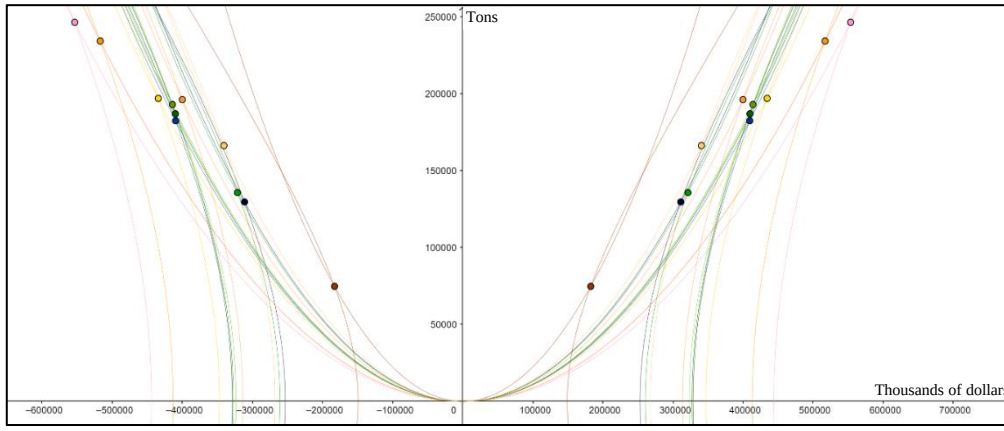

Figure 4. Modeling based on past trading periods
Source: created by the authors using GeoGebra

The points shown in Figure 4 represent observations from previous periods. They are located at the intersection of the conics, and these are the possible trajectories that trade could have followed had commercial conditions been maintained.

Each equation is used to obtain the value, i.e., given a number of goods to be traded, even in real time (e.g., as recorded by customs). Through each equation, the monetary value would be obtained and, consequently, by estimating the unit prices (monetary value/quantity of goods), the minimum, maximum, median, first, and third quartiles are identified. With these, two reference intervals are established (Arteaga, Granados, & Joya, 2013), one consisting of the results of the hyperbolas and the other of those of the parabolas.

It is important to point out that the aim of this work should be clear: to present reference intervals, which, if automated, could be obtained in real time to pre-evaluate operations that suggest price distortions, not to estimate future values as in the case of regressions.

Below are data to obtain reference prices for a commodity[4]. This information corresponds to total exports of Brazilian coffee, compiled on a monthly[5] basis from June 2018 to May 2019 (International Trade Centre, 2019).

Table 1
Worldwide exports of Brazilian coffee, including roasted or decaffeinated coffee, coffee husks, and hulls

Month/year	Thousands of dollars	Tons	Unit Price
June 2018	311 149.00	129 563.33	2.40
July 2018	182 804.00	74 738.42	2.44
August 2018	321 347.00	135 646.29	2.36
September 2018	403 362.00	179 861.20	2.24
October 2018	434 313.00	196 814.51	2.20
November 2018	517 099.00	234 076.34	2.21
December 2018	553 516.00	246 239.32	2.25
January 2019	409 541.00	182 270.22	2.25
February 2019	409 903.00	186 799.18	2.19
March 2019	414 125.00	192 855.77	2.15
April 2019	340 633.00	166 181.64	2.05
May 2019	400 000.00	196 012.56	2.04

Chapter, heading, tariff subheading: 09 01 11 (International Trade Centre, 2019).
Source: created by the authors with data from INTRACEN

With the above data, an additional point—REAL TIME, which is dynamic and with its own conics—is added to the model in Figure 4 to evaluate various quantities of merchandise by verifying the

[4]It can be applied to a large amount of information, including the total number of codes that constitute the Harmonized Commodity Description and Coding System.
[5]In the example, monthly information was included; however, it may be included daily, quarterly, or yearly.

prior initiation of exhaustive customs powers and leaving the latter only to cases that suggest anomalous prices. The information simulated in real time is the record of actual trade at the close of 2019, in which a total of 202 315 tons were exported with a value of 424 000 000 dollars (International Trade Centre, 2020).

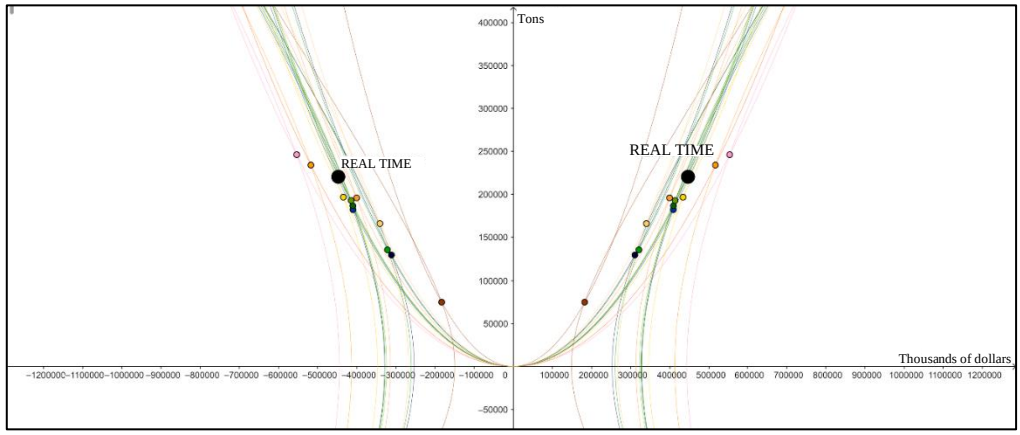

Figure 5. Dynamic model to evaluate the export of Brazilian coffee
Source: created by the authors using GeoGebra

Table 2
Differences between actual price and reference price in thousands of dollars

Months	Reference price		Actual Unit Price as of December 2019	Difference (error)	
	Hyperbola	Parable		Hyperbola	Parable
January	2.11	2.13	2.09	-0.02	-0.04
February	2.09	2.11	2.09	-0.04	-0.02
March	2.08	2.10	2.09	0.01	-0.01
April	1.83	1.86	2.09	0.26	0.23
May	2.00	2.01	2.09	0.09	0.08
June	1.88	1.92	2.09	0.21	0.17
July	1.59	1.92	2.09	0.50	0.17
August	1.89	1.94	2.09	0.20	0.15
September	2.09	2.11	2.09	-0.04	-0.02
October	2.17	2.18	2.09	-0.08	-0.09
November	2.44	2.38	2.09	-0.35	-0.29
December	2.57	2.48	2.09	-0.48	-0.39

Source: created by the authors

The actual record of tons and value in thousands of dollars at the end of 2019 is included (Figure 5). The results obtained through the conics of previous periods are presented in Table 2, including

differences or errors in the last two columns when comparing the real prices for the prices that would have been obtained through the commercial behavior pattern of previous periods.

With the reference prices in Table 2, the following intervals are formed.

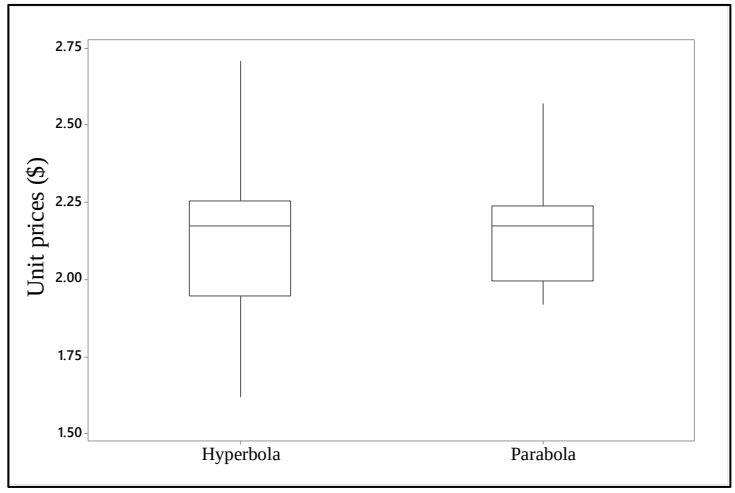

Figure 6. World reference price intervals for Brazilian coffee
Source: created by the authors using Minitab17

Figure 6 shows that, according to hyperbolas, the lowest acceptable price for each ton of coffee would be 1.59, considering the first quartile 1.89, and the median 2.09 with a maximum acceptable value of 2.57. According to parabola estimates, the minimum price would be 1.86, according to the first quartile 1.94, with the median 2.11, and with a maximum acceptable price of 2.48.

Furthermore, if the historical unit prices contained in Table 1 are considered, as well as the fact that coffee cannot be considered volatile due to its low elasticity in demand (Martinez & Salinas, 2004), it is easily identified that the interval from the hyperbola models (black color) would contain 100% of all historical values, which in a general sense would establish the adjustment of the proposal.

Conclusions

In explaining the laws of supply and demand, as well as price elasticity, economics has traditionally adopted the use of curves to represent the displacement of prices and quantities of goods or services. This includes, of course, those traded internationally. Nevertheless, with a more far-reaching vision, curves or conics—specifically hyperbolas and parabolas with different mathematical forms of notation and figure

but with the same objective as the linear models—constitute mathematically valid methods to obtain expectations.

It can be said that the adoption of curves in the economy is due to the question of their origin, i.e., that economic agents will not necessarily obtain an equivalent decrease in the quantity demanded when increasing a price by a certain proportion (a quasi-linear behavior). This is because the value or utility, understood as the purpose of acquiring a good or service (some for use, others for resale), is not always shared since it is influenced by fashion or custom, and these characteristics are well modeled using conics. These offer trajectories of what could be called "commercial behavior," sometimes with greater or lesser curvature, according to the behavior pattern of economic agents and the value they assign to certain products or services.

Hence, the proposals presented in this paper, using conic forms, are not only different from the models that expect centralized or linearized data, such as regressions. They are considered here in their original geometric form, representing the behavior of supply and demand in certain markets with certain products, steepening or contracting their curves in an adaptation of the pattern of behavior in the markets.

It is then that the conics and their properties provide quality of adjustment for complex economic situations, which cannot (and should not), in most cases, be estimated under linear criteria. The reason is that buyers or sellers in the international market must adjust to public policies and their own interests, imposing limits or granting powers, but some of them are only applicable in their place of residence so that all participants do not share the same conditions in international trade.

Under these circumstances, it is possible to identify patterns or trajectories representing how world trade develops at different times, which constitute elements of strength to prevent or pre-evaluate international operations that suggest price distortions. On the one hand, this helps tax and customs authorities to optimize public resources by verifying or auditing operations with higher risk, or by requiring individuals to prove transfer prices, with greater emphasis on operations that suggest a higher risk of distortion.

The possibility of application to evaluate international operations can undoubtedly be extended, even to all 99 HCDS chapters or their breakdown of headings and subheadings. It should be mentioned that hyperbolas provide the best adjustment in the estimates for products with more stable prices. However, for those products or services that are more volatile or have a significant elasticity incidence, the recommendation would be to use the estimates obtained by parabolas.

Service transactions between related parties could be evaluated similarly to goods, choosing the most important transactions at the discretion of the tax authorities or, according to the business sector's interest, when damage or threat of damage to a certain type of service in a local market is identified.

For the estimates of this work, a set of conical models was developed, forming a prototype to evaluate twelve-month operations. Notwithstanding, this prototype can be adapted to either longer periods (quarters or years) or shorter ones (days or weeks). The frequency can also be extended, not being subject to twelve observations but to any other number of time periods. For this reason, if these preventive evaluation measures are adopted, the general suggestion is that the implementation should be coordinated among the different customs offices and with the inspection authorities of the countries concerned, so that there is correspondence in terms of results and detection of anomalies.

Finally, it should be added that the proposals described in this paper are not intended to supplant the methods for determining the customs value of Mexico or any other country, nor to supplant the methods for determining transfer prices. On the contrary, the intention is to complement the conventional mechanisms with prior automatic evaluations that suggest international prices outside of previously observed patterns. In this way, both tax authorities and multinational companies could carry out pre-assessments to verify the validity of international prices.

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