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Valuation of options on Mexico City temperature indices

Valuación de opciones sobre índices de la temperatura de la Ciudad de México

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Abstract

In this work are proposed the innovations to option's pricing on temperature indices assuming that a company needs a hedge, the proposed model is obtained by applying a stochastic differential equation with α-stable processes, analyzing the behavior of the Tacubaya central observatory temperatures during the period from 01/01/1958 to 12/31/2018, estimating descriptive statistics, a significant deterministic mean reversion model and proposing a stochastic mean reversion model with α-stable processes, carrying out a monthly analysis of temperatures, estimating the α-stable parameters and justifying the relevance of the α-stable distributions with goodness of fit tests, estimating the mean reversion parameter to European call option's pricing on temperature indices with α-stable processes and quantify the hedge, concluding that the α-stable options are significant, minimize costs for energy consumption and have a similar behavior to the Gaussian options but with a lower cost.

JEL Code: C46, G13, G32
Keywords: the α-stable processes; weather derivatives; financial engineering

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Resumen

En este trabajo son presentadas innovaciones para valuar opciones sobre índices de temperatura suponiendo que una organización necesita una cobertura, el modelo propuesto es obtenido aplicando una ecuación diferencial estocástica con procesos α-estables, analizando el comportamiento de las temperaturas del observatorio de Tacubaya en el periodo de 01/01/1958 al 31/12/2018, estimando los estadísticos descriptivos, un modelo determinista de reversión a la media significativo y proponiendo un modelo estocástico de reversión a la media con procesos α-estables, realizando un análisis mensual de las temperaturas, estimando los parámetros α-estables y justificando la pertinencia de las distribuciones α-estables con pruebas de bondad de ajuste, estimando el parámetro de reversión a la media para valuar opciones europeas sobre índices de temperatura con procesos α-estables y cuantificar la cobertura, concluyendo que las opciones α-estables son significativas, minimizan los costos por consumo energético y presentan un comportamiento similar a las opciones gaussianas pero con un costo menor.

Código JEL: C46, G13, G32
Palabras clave: procesos α-estables; derivados climáticos; ingeniería financiera

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Introduction

Let it be supposed that a company in the Miguel Hidalgo district of Mexico City needs a hedge for the cost of energy consumption for heating or cooling by replicating the long position in the current value that the company is willing to pay for the temperature index and the long position in a European call option on the temperature index, that is, a hedging strategy concerning the cost of energy that indicates the amount that the company saves and that minimizes the cost of energy consumption derived from the temperature knowing the value of the hedge. Therefore, the valuation of temperature options can optimize energy consumption costs (given the correlation between energy prices) for heating or cooling. The balance between energy demand and supply determines the capital cost of heating or cooling, and energy prices depend on the weather (heating energy demand increases in the fourth quarter due to cold weather, and demand decreases at the end of winter due to warm weather). The portfolio presents two dimensions of capital cost engineering policy: setting the hedging period and taking a long position in the European option on the temperature index to hedge the capital costs for energy consumption that the company is unwilling to hedge. The portfolio quantifies the capital cost per energy consumption that the company saves, minimizing the capital cost per energy consumption by managing risk with the long position in the European call option on the temperature index to minimize the energy cost bill for heating or cooling. The capital cost of energy consumption and the efficiency of the cost policy depends on the valuation of the European call option on temperature. The capital cost of energy consumption requires efficient hedging, which allows companies to monitor consumption while preserving competitiveness. Energy dependence derived from temperature represents an operational risk, and efficient hedging minimizes the capital cost of energy consumption, facilitating growth and economic development. Hedges drive preferences toward valuations, which exhibit relevance and meaningful prices.

Weather impacts companies' financial and economic situation in different economic sectors (primary: agriculture, forestry, livestock, mining, and fishing; secondary: construction and manufacturing; and tertiary: commerce, services, and transportation). Financial engineering through weather derivatives is a tool used in risk management for hedging potential losses caused by weather. The Chicago Mercantile Exchange (CME) is a pioneer in negotiating weather derivatives where the temperature is an explanatory variable of energy consumption and derived products or services. The Chicago Board of Trade (CBOT) negotiated the first commodity options in 1984 and introduced catastrophe options. Therefore, it is important that in Mexico, there is the possibility of risk hedging in the context of costs derived from weather changes because they are potentially significant for the economy as a result of the energy reform. In order to trade weather derivatives, it is necessary to identify risks to be hedged and valuation models and accept the impossibility of completely transferring risks; therefore, incomplete markets are adequate because they accept proposals for indices and models, innovating through financial engineering in hedging and managing weather risks.

Considine (2000) indicates that the first transaction in the weather derivatives market was in the United States in 1997. Jain and Baile (2000) indicate that approximately 3 000 contracts worth USD 5.5 billion have been completed in the U.S. weather derivatives market, and about 100 weather contracts worth GBP 30 million have been completed in Europe. Brody et al. (2002) indicate that temperature is modeled with Brownian motion and characterize the temperature dynamics with an Uhlenbeck and Ornstein (1930) process for pricing two types of contingent claims, one based on degrees of heating and cooling and the other based on cumulative temperature, and propose analytical expressions for the expected discounted payoffs of these derivatives.

Alaton et al. (2002) indicate that weather impacts business activities. The energy sector has driven weather derivatives and the progress of weather risk management; the underlying variables are: temperature, humidity, rain, or snowfall and the common underlying is temperature; energy producers have recorded that prices are correlated with weather and weather derivatives are a way to hedge risks; capital markets and the insurance sector have come together, and the number of catastrophe bonds issued has increased; the CBOT has introduced catastrophe options, and weather derivatives are the logical extension; weather derivatives allow costs arising from weather changes to be hedged. The authors model prices for weather derivatives with contingent payments as a function of temperature using historical data to suggest a Brownian stochastic process describing temperature, obtaining prices in an incomplete market. They present numerical results using an approximation formula and Monte Carlo simulations. They conclude that the model implemented is a simplification of reality and could be improved and indicate that even though the model seems to fit the temperature data well and the model is close to reality, applying a model for the dynamic volatility process for significant forecasts in the valuation of weather derivatives and with real-time trading could find an adequate structure for market prices.

Benth (2003) proposes arbitrage-free price dynamics for temperature-contingent claims with an Uhlenbeck and Ornstein (1930) process and proposes explicit expressions for European and average-type contingent claims. Platen and West (2004) propose an approach for pricing weather derivatives traded in an incomplete market. They show that the actuarial methodology is a particular case of the concept of fair prices. They propose a discrete weather model to approximate the historical characteristics of the climate. Benth et al. (2011) indicate that weather derivatives differ from financial derivatives because the weather cannot be traded and therefore is not replicated by other financial instruments. They analyzed the market price structure in emerging Asian markets and found that Asian temperatures (Tokyo, Osaka, Beijing, Teipei) are Gaussian in the sense that stochastic processes are close to a Wiener process. Manfredo and Richards (2009) indicate that weather derivatives are a financial innovation for risk management. When weather derivatives are used to hedge risks, risk managers face hazards arising from the choice of the weather station where a derivative contract is entered into and the underlying weather index. They show that the nonlinear relation between crop returns and weather creates a specific hedge for weather options.

Musshoff et al. (2009) indicate that weather is a source of crop uncertainty, climate fluctuations will increase in the future due to climate change, and farmers will try to protect themselves against weather-related variations in yields through insurance, and that recently there has been a discussion on the use of weather derivatives to hedge volumetric risks. Even though weather derivatives show advantages over insurance, there is a small market for these products in agriculture. Using stochastic simulation, they analyze actual yields and weather data from northeastern Germany to quantify the risk reduction achieved in wheat production with rainfall options.

Nave Pineda and González Sánchez (2010) indicate that economic activities are exposed to climatic conditions and are relevant to the energy, agricultural, tourism and insurance sectors. They surmise that demand for weather hedging products is growing, driving weather derivatives and the development of weather risk management. They suggest that weather derivatives markets call for standardization of contract terms. They mention that temperature derivatives are the most widely used because temperature is an explanatory variable for energy consumption. They analyze temperature derivatives applied to Eldorado Airport, adjusting the temperature to an Uhlenbeck and Ornstein (1930) mean-reverting model with a Wiener process, test its validity with simulation, and perform the valuation of several derivatives. They conclude that weather derivatives hedge the income statements of economic activities sensitive to weather factors. Their presence in financial markets confers flexibility in the risk management of business activity in general and in the insurance and electricity sectors. The development and use of financial derivative products require the analysis of the underlying factors for an adequate measurement and valuation of the risk to be managed. The Uhlenbeck and Ornstein (1930) model presents an adequate goodness of fit for the 28-year daily data sample (1979-2006), confirming the properties of mean reversion and seasonal volatility for monthly periods using the model for the valuation of temperature derivative products with simulation techniques and closed formulas that assume a Gaussian behavior of monthly temperatures.

Alva Vázquez and Sierra Juárez (2010) point out that due to Mexico's geographic location, various natural phenomena occur yearly, generating changes that cause disasters (earthquakes, hurricanes, floods, and droughts) and affect natural and financial resources. "El Niño" affects fisheries in monetary terms, and its effects meant a loss of approximately USD 700 000 000 in 1998. The companies exposed to climate risk include energy producers and consumers, supermarket chains, amusement and recreation industries, and agricultural and fishing industries. They indicate that in Mexico, there is no market for climate derivatives and propose a model for the valuation of climate options for the Mexican Pacific Ocean fishing sector, where the essential variable is sea temperature, using a hedging system with historical data on sea temperature in different regions to design a stochastic process that describes the evolution of ocean temperature. They conclude that option premiums are within the range of 10 to 20% of the contract's notional value. Therefore, the existence of weather options is useful for hedging against weather changes in the oceans that affect the environment, adapting the forecasts of fundamental indices (temperature, rain, and snow) used in weather derivatives.

Goncu (2011) notes that weather derivatives are a form of financial security with contingent payments conditional on weather indices that provide hedges for the risk of weather changes in terms of revenues and costs and that there is meaningful potential for their use because approximately one-seventh of the industrial sector is weather-sensitive. He explains that agriculture is significantly affected by climatic variables. He analyzes weather derivatives based on the Heating Degree Days (HDD) and Cooling Degree Days (CDD) temperature indices traded on the CME. He indicates that this is the first study for pricing weather derivatives based on the daily average temperatures of Beijing, Shanghai, and Shenzhen using a dynamic model with a period-constant volatility function, where option price estimates are obtained with Monte Carlo simulation and analytical approximation methods. He concludes that HDD and CDD options exhibit lower relative error for the Monte Carlo estimator relative to the analytical approximation as options become more in-the-money and that the approximation formula is less reliable for HDD contracts in Shenzhen because the distribution assumption is breached by empirical observations on some occasions, generating larger relative errors. When the underlying distribution assumption is breached, the Monte Carlo simulation and approximation produce close results.

Chang and Tang (2016) indicate that weather options enable companies to insure against weather fluctuations. Assuming self-financing and a correlated set following a geometric Brownian motion with a jumping diffusion process, they present a model for the valuation of climate options on temperature with a Brownian mean-reverting motion, deriving a two-dimensional partial differential equation by applying a numerical method based on a finite volume technique combined with the Lagrangian derivative and provide numerical examples for a series of European put options on the warming degree indices. Tang and Chang (2016) present a model for the valuation of climate options on temperature that follows a mean reversion motion with a Brownian motion of diffusion with jumps assuming self-financing and deriving a two-dimensional partial differential integer equation for the valuation of a series of European put options on the warming degree index by applying numerical techniques.

Groll et al. (2016) discuss a methodology for valuing temperature derivatives with forward-looking information providing empirical support for the theoretical framework of consistent models for temperature forecasts and apply this methodology by performing a statistical analysis of weather forecasts and propose a two-factor Uhlenbeck and Ornstein (1930) model with two independent Gaussian processes considering forward-looking information, valuing temperature derivatives and calibrating the market risk price. The model's power is compared to alternative models, confirming that the market risk price is due to information misspecification.

Huang et al. (2018) investigate temperature behavior to develop an ARFIMA Seasonal GARCH model that models seasonal, cyclical, global warming, and long-memory effects that provide goodness of fit and forecast accuracy using daily average temperatures in six U.S. cities. They analyze temperature behavior in temperature derivative prices and propose numerical valuation of options for forward warming and cooling indices and option contracts with the developed model that contributes to the development for weather derivatives markets.

Dzupire et al. (2019) indicate that weather derivatives markets are incomplete because weather indices are not tradable assets. Therefore, valuation methods such as Black and Scholes (1973) are not applicable in the valuation of weather derivatives, so they develop a method for the valuation of a basket of weather derivatives on rainfall that is modeled as a stationary Gamma process transformed into a geometric Brownian process on temperature that is modeled with an Uhlenbeck and Ornstein (1930) process with a Wiener process correlated with that of rainfall in an incomplete market. The risk preference of the economic agent is an exponential utility function. Prices are derived with the dynamic programming principle starting from stochastic optimal control problems, and they find that the indifference measure is equal to the physical measure because the correlation between the capital market and the weather is nonexistent. The fair price of the derivative is higher than the seller's indifference price and lower than

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the buyer's indifference price due to the market's viability without arbitrage opportunities. They conclude that the developed model can apply a different profit function, which means a different risk preference, and since weather derivatives are traded over the counter, the valuations are compared with actuarial approaches.

The hypothesis proposed in this paper is that it is possible to innovate in the modeling of temperature for the valuation of derivative products to hedge costs derived from the risks of temperature changes. The objective is to propose a model for the valuation of options on temperature and test the suitability of the model through distributions that model extreme and asymmetric events that generate financial and economic impacts of higher amounts than those expected by the Uhlenbeck and Ornstein Gaussian model (1930), and which also satisfy the generalized central limit theorem in α-stable domains of attraction that more adequately estimate project risks with financial engineering and risk management using the theory of valuation of weather derivatives in incomplete markets with the stability parameter.

The paper is organized as follows: in section 2, the seasonal behavior of temperatures is described, descriptive statistics are estimated, and a qualitative analysis of the empirical distribution is presented. A simple linear regression model with a positive and significant trend is estimated, and a deterministic mean reversion model with ordinate to the origin, positive trend, and meaningful amplitude improves the simple linear regression model. Also, a stochastic mean-reversion model with a stochastic α-stable process is proposed, a monthly analysis of the temperatures is performed, the monthly α-stable parameters are estimated, the goodness of fit tests are performed, and the α-stable distributions for the monthly mean temperatures are not rejected. The mean-reversion parameter estimated with stochastic α-stable processes is proposed, and the solution of the stochastic differential equation is proposed. Section 3 proposes the model for the valuation of European call options on the heating and cooling degree indices with α-stable distributions, the 2010 temperature indices are estimated, and the call options on the indices are valued. Section 4 presents the conclusions, followed by the bibliographical references.

Average temperature modeling

Temperature is an exogenous factor affecting weather derivatives' valuation to cover costs arising from temperature changes. The data used are the daily minimum, maximum and average temperatures from the Tacubaya Central observatory (CNA-SMN-SCDI, statistical climatology, unit: °C, station: 9048, state: Mexico City, municipality: Miguel Hidalgo, agency: CONAGUA-DGE, latitude: 9°24'13" North, longitude: 99°11'46" West, altitude: 2 308.6 masl) during the period from 01/01/1958 to 31 /12/2018 with a total of 22 280 observations. The data are used to adjust a model that describes the underlying climate derivatives, proposing that the sectors that use hedges have timely and accurate information that the observatories guarantee so that the existence of indices permits investors and issuers to model, validate, and quantify climate risks with official information for operation in national and international markets. The information from the Tacubaya observatory contains daily minimum and maximum temperatures in degrees Celsius, and from these, the average temperature is calculated as follows:

$$\text{pro}\left(T_k\right)=\frac{\text{mín}\left(T_k\right)+\text{máx}\left(T_k\right)}{2} \tag{1}$$

where $\text{pro}\left(T_k\right)$ is the average temperature, and $\text{mín}\left(T_k\right)$ and $\text{máx}\left(T_k\right)$ are the minimum and maximum temperatures. The behavior of the temperatures is shown in Figure 1.

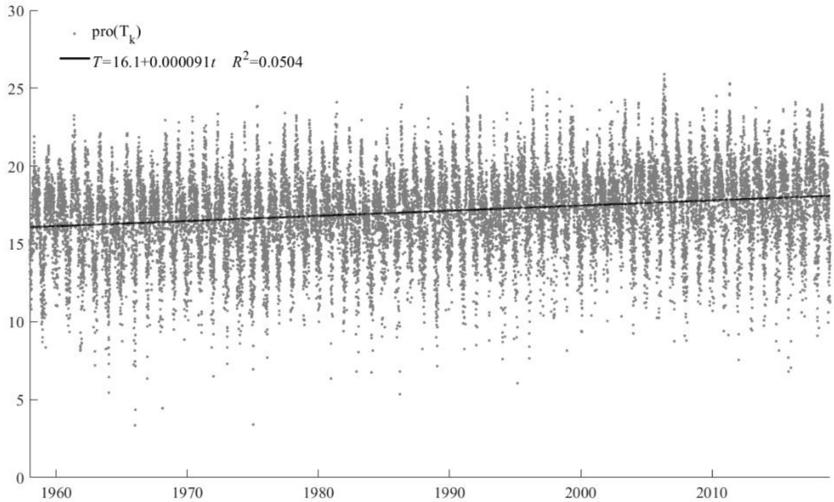

Figure 1. Behavior of average temperatures
Source: created by the authors with data from the Tacubaya central observatory

Figure 1 indicates that average temperatures range from 3.35°C on January 8, 1964, to 25.9°C on May 10, 2006. The coldest periods occur between October and March, and the warmest periods occur between March and June, with an average of 17.08°C, a standard deviation of 2.60°C and a significant positive trend. The estimation of the descriptive statistics of the average temperatures is presented in Table 1.

Table 1
Descriptive statistics of average temperatures

Temperature	Minimum	Maximum	Average	Deviation	Skewness	Kurtosis
Average	3.35	25.90	17.08	2.60	-0.36	3.46

Source: created by the authors with data from the Tacubaya central observatory

Table 1 indicates that the average temperatures range from 3.35°C to 25.9°C with an average of 17.08°C, a volatility of 2.60°C. The negative skewness coefficient indicates that the average temperatures have an asymmetric distribution with a lower slope at the left end than at the right end of the mode $X = 18$, and the kurtosis coefficient greater than three indicates that the distribution of the average temperatures is leptokurtic concerning the Gaussian distribution. Therefore, the temperatures have leptokurtic and asymmetric distributions. The histogram of absolute frequencies of average temperatures is presented in Figure 2.

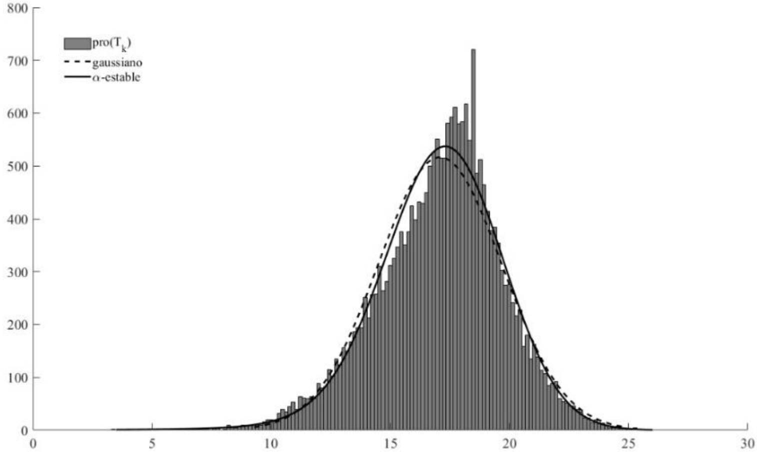

Figure 2. Frequencies of average temperatures
Source: created by the authors with data from the Tacubaya central observatory

Figure 2 indicates that the average temperatures present a distribution with negative skewness; that is, the average temperatures lower than the mode for grouped data, $X = 18.10$, represent 60.76% of the average temperatures and present a leptokurtic distribution concerning the Gaussian distribution, that is, the right and left extremes of the empirical distribution present frequencies higher than those expected by the Gaussian distribution.

Extreme values (minimum or maximum) of average temperatures represent a financial and economic impact higher than the cost of production or service derived from leptokurtosis and skewness because the Gaussian distribution is inappropriate for modeling extreme values and skewness. After all, it is symmetrical and mesokurtic. The climatic events that impact the companies and that the Gaussian distribution is unable to model adequately indicate the importance of valuing appropriate hedges in the face of climatic changes caused by exogenous factors, including human activities such as agriculture, stockbreeding, and deforestation. It is also important to assess the use of fossil fuels such as coal, oil, gasoline, diesel, aviation fuel, and gas, which generate atmospheric and biophysical changes that produce greenhouse gases such as methane (NH_4), carbon oxide (CO_2), ozone (O_3), and nitrogen oxides (NO, NO_2, N_2O, N_2O_3, N_2O_4, N_2O_5). Furthermore, factors such as solar radiation are outside the analysis and objectives of this work, which is focused on the valuation of options on temperature indices, an underlying that presents a significant positive trend in recent years (1958-2018). The estimates of the ordinate to the origin and the slope of the average temperatures are presented in Table 2.

Table 2
Estimation of the ordinate and slope of average temperatures

Temperature	β_0	β_1	R^2	$P\left(t_{\beta_0}\right)$	$P\left(t_{\beta_1}\right)$	$P\left(F\right)$
Average	16.1	0.000091	0.0504	0.0000	2.64E-252	2.64E-252

Source: created by the authors with data from the Tacubaya central observatory

Table 2 indicates that the ordinate of the average temperatures is 16.11°C and the slope is 0.000091, and they are represented by the equation and the solid line in Figure 1; the forecast is 18.11°C by the end of 2018; the coefficient of determination is 0.0504. The probabilities of the t and F statistics are significant for the average temperatures; therefore, the temperature increases are significant and an increase of 2.04°C is expected.

Deterministic mean-reversion model

Figure 1 shows a periodic variation of temperatures with annual periods, with positive and significant trends, and with exogenous factors as causal; therefore, a deterministic mean-reverting model is estimated to model the temperatures:

$$T_t^m = A + Bt + C\,\mathrm{sen}\left(\omega\, t + \varphi\right) \tag{2}$$

where T_t^m is the temperature, t represents the time, and the sine function models the annual periodic variation, therefore, $\omega = 2\pi(365)^{-1}$, φ represents a phase angle for the cold and warm periods occurring at different dates at the beginning or end of the year, A, B, C and φ are the estimators that adjust the temperatures through the least squares method, using the identity:

$$\|(a,b)\| \operatorname{sen}\left(\omega\, t + \arctan\left(ba^{-1}\right)\right) = a \operatorname{sen}(\omega\, t) + b\cos(\omega\, t)$$

thus, the transformed model is:

$$T = \beta_0 + \beta_1 t + \beta_2 \operatorname{sen}(\omega\, t) + \beta_3 \cos(\omega\, t) \tag{3}$$

and the parameters for the average temperatures are:

$$A = \beta_0, \quad B = \beta_1, \quad C = \sqrt{\beta_2^2 + \beta_3^2}, \quad \varphi = \arctan\left(\beta_3 \beta_2^{-1}\right) \tag{4}$$

where A is the ordinate, B is the slope, C is the amplitude of the annual periodic variation function, and φ is the phase angle of the cold and warm periods. The estimation of parameters of the deterministic mean reversion model through Equation is presented in Table 3.

Table 3
Parameter estimation of the transformed model

Temperature	β_0	β_1	β_2	β_3	R^2	$P\left(t_{\beta_0}\right)$	$P\left(t_{\beta_1}\right)$	$P\left(t_{\beta_2}\right)$	$P\left(t_{\beta_3}\right)$	$P(F)$
Average	16.1	0.092E-5	0.8	-2.2	0.46	0.00	0.00	0.00	0.00	0.00

Source: created by the authors with data from the Tacubaya central observatory

Table 3 indicates that the ordinate of the deterministic mean reversion model is 16.1°C and is close to the ordinate of the simple linear regression model; the slope is 0.000092; the slopes, β_2 and β_3, to calculate the amplitude of the periodic variation, are significant; the coefficient of determination is 0.46. Thus the deterministic mean reversion model improves the simple linear model and the probabilities of the statistics F indicate that the deterministic mean reversion model is significant. The parameter estimates of the deterministic mean reversion model, with Equations and are presented in Table 4.

Table 4
Parameter estimation of the deterministic mean-reversion model

Temperature	A	B	C	φ
Average	16.1	0.000092	2.35	-1.22

Source: created by the authors with data from the Tacubaya central observatory

Table 4 indicates that the ordinate of the deterministic mean-reversion model is 16.1°C; the slope is 0.000092; the amplitude of the periodic variation function is 2.35, i.e., the difference between average temperatures is 4.7°C. The results of the diagnostic tests for the residuals of the deterministic models are presented in Table 5.

Table 5
Diagnostic tests of deterministic models

		$T=\beta_0+\beta_1 t$		$T=\beta_0+\beta_1 t+\beta_2\,\mathrm{sen}(\omega\, t)+\beta_3\cos(\omega\, t)$		
Test	Statistical	Value	$P\ (T>t)$	Statistical	Value	$P\ (T>t)$
Correlation	$\chi^2_{LM}(1)$	16 927.6	0.0000	$\chi^2_{LM}(1)$	1 3375.3	0.0000
Correlation	$\chi^2_{LM}(2)$	16 941.4	0.0000	$\chi^2_{LM}(2)$	13 375.3	0.0000
Heterosced asticity	$\chi^2_{BPG}(1)$	0.0	0.9787	$\chi^2_{BPG}(3)$	419.1	0.0000
Heterosced asticity	$\chi^2_{W}(2)$	20.5	0.0000	$\chi^2_{W}(9)$	558.1	0.0000
Specificatio n	$F(1,n-k-1)$	3.4	0.0649	$F(1,n-k-1)$	493.0	0.0000
Normality	$\chi^2_{JB}(2)$	760.1	0.0000	$\chi^2_{JB}(2)$	1 190.5	0.0000

Source: created by the authors with data from the Tacubaya central observatory

Table 5 presents the results of the misspecification tests applied to the estimated models. The tests applied are Breusch and Godfrey tests for serial correlation with $\chi^2_{LM}(1)$ and $\chi^2_{LM}(2)$, Breusch, Pagan and Godfrey and White for heteroscedasticity with $\chi^2_{BPG}(k-1)$ and $\chi^2_{W}(l)$, Ramsey for specification with $F(1,n-k-1)$, and Jarque and Bera for normality with $\chi^2_{JB}(2)$. The results indicate that the simple linear model passes the Breusch and Godfrey tests for serial correlation and heteroscedasticity at 5%. The deterministic mean reversion model does not pass any of the applied tests. The results are attributed to the inherent volatility of daily temperatures, which is very volatile in some periods, to structural temperature changes, or regime shifts such as climate change. The deterministic mean-reversion model from January 1, 2009, to December 31, 2018, is presented in Figure 3.

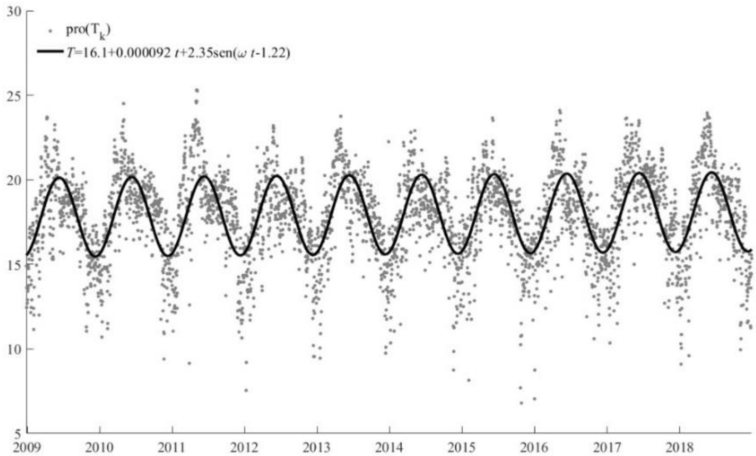

Figure 3. Deterministic model of reversion to mean temperatures
Source: created by the authors with data from the Tacubaya central observatory

Figure 3 presents the fit of the deterministic mean-reverting model with the average temperatures observed during the last ten years, and it is observed that it captures more adequately the annual oscillation period and the positive trend than the simple linear model. Therefore, there is a deterministic mean-reversion model to model temperatures.

Stochastic mean reversion model

The behavior of the temperatures presents a periodic variation with a positive trend and a stochastic variation around the periodic variation. Therefore, the first innovation presented is the stochastic differential equation:

$$\mathrm{d}T_t = a\left(T_t^m - T_t\right)\mathrm{d}t + \gamma_t\,\mathrm{d}Z_t \tag{5}$$

where a is the rate of reversion to the mean, γ_t is the scaling parameter, and Z_t is a standard, self-similar α-stable process. The solution of the Equation, in the particular case where Z_t is a Wiener process, is an Uhlenbeck and Ornstein (1930) process. In the long run, the Equation shows no reversion to the mean, hence, including the derivative:

$$\frac{\mathrm{d}T_t^m}{\mathrm{d}t} = B + \omega\ C\cos\left(\omega\ t + \varphi\right)$$

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to the trend of the Equation, reversion to the mean is obtained because the temperatures modeled by T_t^m present periodic variations that adjust the trend so that the stochastic differential equation presents reversion to the mean in the long term. With the initial condition $T_s = x$ a stochastic differential equation is obtained to model the temperature:

$$\mathrm{d}T_t = \left(\frac{\mathrm{d}T_t^m}{\mathrm{d}t} + a\left(T_t^m - T_t\right) \right) \mathrm{d}t + \gamma_t \, \mathrm{d}Z_t \quad \forall t > s \tag{6}$$

with the solution:

$$T_t = \left(T_s - T_s^m\right)\exp\left(-a\left(t-s\right)\right) + T_t^m + \int_s^t \exp\left(-a\left(t-\tau\right)\right)\gamma_\tau \, \mathrm{d}Z_\tau \tag{7}$$

where $T_t^m = A + Bt + C\,\mathrm{sen}\left(\omega\, t + \varphi\right)$.

Monthly analysis of average temperatures

Based on the previous results, a monthly analysis of average temperatures is performed. The α-stable parameter estimates for the monthly periods are presented in Table 6.

Table 6
Estimation of α-stable parameters of monthly temperatures

Period	α	β	γ	δ
January	1.78	-1.00	1.26	14.1
February	1.84	-1.00	1.41	15.5
March	1.84	-1.00	1.38	17.8
April	2.00	0.28	1.52	19.2
May	2.00	0.54	1.47	19.6
June	2.00	0.10	1.24	18.9
July	1.97	-1.00	0.93	17.8
August	1.97	-1.00	0.81	18.0
September	1.82	-1.00	1.03	17.4
October	1.82	-0.99	1.32	16.5
November	1.78	-1.00	1.26	15.3
December	1.85	-1.00	1.25	14.2

Source: created by the authors with data from the Tacubaya central observatory

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Table 6 presents the α-stable parameters. The stability parameters indicate that the distributions of the monthly mean temperatures are leptokurtic, except for May, June, and August for statistic A^2 and April through August for statistic W^2. The skewness parameters indicate that the distributions spread to the far left more slowly than to the far right, except for April, May, and June. The scaling parameters indicate that the distributions are seasonally dispersed. The location parameters indicate that the distributions show seasonal changes. Concluding that monthly temperatures are leptokurtic and negatively skewed in most months, it is necessary to test the relevance of α-stable distributions for modeling monthly temperatures. Additionally, the quantitative analysis to test the hypothesis about monthly temperatures presenting Gaussian distributions with the Anderson and Darling (AD) and Cramér von Mises (CvM) goodness of fit tests is presented in Table 7.

Table 7
AD and CvM tests for Gaussian distributions

Period	A^2	$P\left(A^2 > a^2\right)$	W^2	$P\left(W^2 > w^2\right)$
January	11.37	0.0005	1.82	0.0000
February	6.92	0.0005	1.15	0.0010
March	7.51	0.0005	1.15	0.0000
April	2.75	0.0005	0.38	0.0760
May	0.44	0.2911	0.07	0.7260
June	0.59	0.1278	0.09	0.6530
July	1.12	0.0062	0.21	0.2800
August	0.73	0.0574	0.12	0.4730
September	8.75	0.0005	1.38	0.0000
October	9.77	0.0005	1.63	0.0000
November	10.27	0.0005	1.69	0.0000
December	8.19	0.0005	1.43	0.0000

Source: created by the authors with data from the Tacubaya central observatory

Table 7 presents the statistic A^2 and the significance level of the AD test. Therefore, the hypothesis that the temperatures present Gaussian distributions in 9 of the 12 monthly periods studied is rejected, that is, 75%, except for the average temperatures of May, June, and August; therefore, the Gaussian hypothesis for modeling the temperatures is unjustifiable. The statistic W^2 and significance level of the CvM test rejects the hypothesis that temperatures present Gaussian distributions in 7 of the 12 monthly periods studied, i.e., 58.33%, except for the average temperatures from April to August; therefore, the Gaussian hypothesis for modeling temperatures is also unjustifiable. The quantitative analysis to test the hypothesis that monthly temperatures present α-stable distributions with AD and CvM goodness of fit tests is presented in Table 8.

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Table 8
AD and CvM tests for α-stable distributions

Period	A^2	$P\left(A^2 > a^2\right)$	W^2	$P\left(W^2 > w^2\right)$
January	0.46	0.7831	0.05	0.8550
February	1.58	0.1585	0.25	0.1780
March	0.27	0.9584	0.03	0.9600
April	2.75	0.0368	0.38	0.0670
May	0.44	0.8046	0.07	0.7240
June	0.59	0.6609	0.09	0.6700
July	0.62	0.6322	0.11	0.5050
August	0.49	0.7551	0.08	0.6700
September	0.49	0.7598	0.06	0.7950
October	1.52	0.1725	0.26	0.1910
November	1.11	0.3035	0.18	0.3040
December	1.69	0.1375	0.29	0.1350

Source: created by the authors with data from the Tacubaya central observatory

Table 8 presents the statistics A^2 and W^2 and the significance levels of the AD and CvM tests; therefore, the hypothesis that temperatures present α-stable distributions is not rejected in the totality of the 12 monthly periods studied, i.e., 100%; therefore, the application of α-stable distributions is justified to model monthly temperatures. Given the results presented in Tables 6, 7, and 8, it is concluded that it is feasible to model temperatures and propose a model to value monthly options on temperature indices in incomplete markets with α-stable distributions to generate annual hedges.

Estimation of the mean-reversion parameter

The observations of the temperatures are daily; therefore, to estimate the mean reversion parameter, an efficient estimator is proposed assuming that the scale parameter is constant in monthly periods, therefore:

$$G\left(\hat{a}\right) = \sum_{k=1}^{n} \frac{T_k^m - T_k}{\gamma_{k_m}^{\alpha}} \left(T_{k+1} - T_{k+1}^m - \left(T_k - T_k^m\right)\exp\left(-a\right)\right) = 0 \tag{8}$$

where γ_{k_m} is the monthly temperature dispersion parameter for $m = 1, 2, \ldots, 12$. Therefore, the second innovation is the estimation of the mean-reversion parameter:

$$\hat{a} = \ln\left(\sum_{k=1}^{n} \frac{T_k^m - T_k}{\gamma_{k_m}^{\alpha}}\left(T_k - T_k^m\right)\right) - \ln\left(\sum_{k=1}^{n-1} \frac{T_k^m - T_k}{\gamma_{k_m}^{\alpha}}\left(T_{k+1} - T_{k+1}^m\right)\right) \tag{9}$$

where $T_t^m = A + Bt + C\,\mathrm{sen}\left(\omega\, t + \varphi\right)$. The estimation of the mean reversion parameter through Equation is presented in Table 9.

Table 9
Estimation of the mean-reversion parameter

Temperature	a
Average	0.264817

Source: created by the authors with data from the Tacubaya central observatory

Table 9 presents the mean-reversion estimator, which indicates the speed with which average temperatures return to the mean. The result is consistent with those of Alaton et al. (2002) and Alva Vázquez and Sierra Juárez (2010). The third innovation, according to Example 3.6.4 of Samorodnitsky and Taqqu (1994), which is presented as the solution to Equation, is:

$$T_t = \left(T_s - T_s^m\right)\exp\left(-a\left(t-s\right)\right) + T_t^m + \gamma_{t_m}\left(\frac{1-\exp\left(-\alpha_{t_m} a\left(t-s\right)\right)}{\alpha_{t_m} a}\right)^{\frac{1}{\alpha_{t_m}}} Z_{t-s} \tag{10}$$

where Z_{t-s} are standardized and independent α-stable random variables; therefore, the proposed stochastic mean reversion model is:

$$T_t = \left(T_{t-1} - T_{t-1}^m\right)\exp\left(-a\Delta t\right) + T_t^m + \gamma_{t_m}\left(\frac{1-\exp\left(-\alpha_{t_m} a\Delta t\right)}{\alpha_{t_m} a}\right)^{\frac{1}{\alpha_{t_m}}} Z_t \tag{11}$$

where $Z_t \sim S_1\left(\alpha, \beta\right)$.

The stochastic mean-reversion model of 2019 temperatures using the average of 30 000 simulations for each day is presented in Figure 4.

http://dx.doi.org/10.22201/fca.24488410e.2021.2722

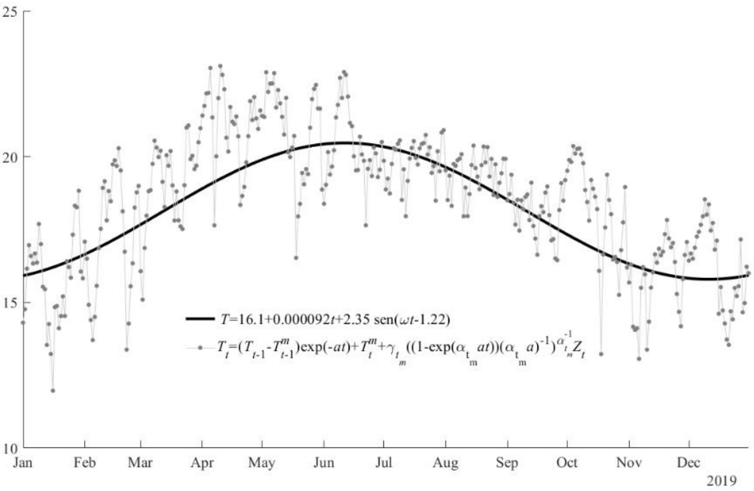

Figure 4. Stochastic model of temperature mean-reversion 2019
Source: created by the authors with data from the Tacubaya central observatory

Figure 4 presents the behavior of the stochastic mean-reversion model using the daily average of 30 000 standardized and independent α-stable random variable simulations for the year 2019, and it is observed that the random process around the deterministic mean-reversion process replicates the temperatures and, where appropriate, allows the simulation of temperatures of the following year.

Weather derivatives

The weather derivatives market provides companies with hedges for risks arising from temperature changes. The energy industry drove the growth of weather derivatives, consummating the trading of weather derivatives at the CME in 1997; therefore, risk hedges must exist in the context of costs derived from weather changes in Mexico. The energy reform generates opportunities to trade weather derivatives in Mexico considering trading levels, risks to be hedged, and valuation models, and accepting the impossibility of completely transferring risks. Therefore, incomplete markets are the appropriate models for indices, innovating with financial engineering: hedging and managing weather risks.

Valuation of call options on temperature

Assuming that the market risk price is constant and that the company buys a risk-free asset that is invested at the constant interest rate i, and they buy a contract that for each degree Celsius pays one monetary unit, then, under an equivalent measure, the pricing process satisfies the Equation. Climate derivatives on temperature are based on daily degrees of warming or cooling. The settlement payment for a European call option on the daily heating index is:

$$H_{\tau} = N \operatorname{máx}\left(H_T - S, 0\right) = N \operatorname{máx}\left(\sum_{k=1}^{n} \operatorname{máx}\left(T_0 - T_k, 0\right) - S, 0\right) \tag{12}$$

where H_{τ} is the contingent payment and is similar to an Asian arithmetic average option, N is the cost per heating degree per day, H_T is the rate of heating degrees per day, and T_0 is the reference temperature, usually 18°C. Therefore, if the average temperature is lower than 18°C, it is necessary to use energy to increase the temperature; therefore, the reference temperature is modified according to the coverage needs. The settlement payment for a European call option on the daily cooling index is:

$$C_{\tau} = N \operatorname{máx}\left(C_T - S, 0\right) = N \operatorname{máx}\left(\sum_{k=1}^{n} \operatorname{máx}\left(T_k - T_0, 0\right) - S, 0\right) \tag{13}$$

where C_{τ} is the contingent payment, N is the cost per degree of daily cooling, C_T is the daily cooling rate, and T_0 is the reference temperature, 18°C. Therefore, if the average temperature is higher than 18°C, it is necessary to use energy to lower it; therefore, the reference temperature is modified according to the hedge needs. The fourth innovation presented is that if the process $H_{\tau} \sim S_1\left(\alpha, \beta, \gamma, \delta\right)$, then $H_{\tau} = H_0 + \gamma Z_{\tau}$ and therefore, the price of the European call option on H_T is:

$$c\left(\tau, H_{\tau}\right) = \exp\left(-i\tau\right) \mathrm{E}\left(\operatorname{máx}\left(H_{\tau} - S, 0\right) \middle| F_{\tau}\right) = \exp\left(-i\tau\right) \int_S^{\infty} \left(H_{\tau} - S\right) f_{Y_{\tau}}\left(y_{\tau}, \alpha, \beta, \gamma\right) \mathrm{d}Y_{\tau}$$

where $\tau = T - t$ is the remaining time, and the Gaussian model $H_{\tau} \sim S_1\left(2, 0, \sigma, \mu\right)$ is a particular case of the α-stable model. Standardizing the distribution function:

$$c(\tau,H_\tau)=\exp(-i\tau)\int_{\frac{S-H_0}{\gamma}}^{\infty}(H_\tau-S)f_{Y_\tau}(y_\tau,\alpha,\beta)\mathrm{d}Y_\tau=\exp(-i\tau)\int_{-d}^{\infty}(H_\tau-S)f_{Y_\tau}(y_\tau,\alpha,\beta)\mathrm{d}Y_\tau$$

Substituting $H_\tau = H_0+\gamma Z_\tau$ and by Theorem 2.4.1 of Zolotarev (1986):

$$c(\tau,H_\tau)=\exp(-i\tau)\int_{-d}^{\infty}(H_0+\gamma Z_\tau-S)f_{Y_\tau}(y_\tau,\alpha,\beta)\mathrm{d}Y_\tau$$
$$c(\tau,H_\tau)=\exp(-i\tau)\left((H_0-S)\int_{-d}^{\infty}f_{Y_\tau}(y_\tau,\alpha,\beta)\mathrm{d}Y_\tau+\gamma\int_{-d}^{\infty}Z_\tau f_{Y_\tau}(y_\tau,\alpha,\beta)\mathrm{d}Y_\tau\right)$$
$$c(\tau,H_\tau)=\exp(-i\tau)\left((H_0-S)\int_{-\infty}^{d}f_{Y_\tau}(y_\tau,\alpha,-\beta)\mathrm{d}Y_\tau+\gamma\int_{-\infty}^{d}Z_\tau f_{Y_\tau}(y_\tau,\alpha,-\beta)\mathrm{d}Y_\tau\right)$$

Therefore, the valuation of European call options over the daily heating index with the α-stable model is:

$$c(\tau,H_\tau)=\exp(-i\tau)\left((H_0-S)\Phi(d,\alpha,-\beta)+\gamma f_{Y_\tau}(d,\alpha,-\beta)\right)$$
$$c(\tau,H_\tau)=\exp(-i\tau)\left((H_0-S)(1-\Phi(-d,\alpha,\beta))+\gamma f_{Y_\tau}(-d,\alpha,\beta)\right) \tag{14}$$

where $d=\dfrac{H_0-S_H}{\gamma}$ and $H_0=\sum_{k=1}^{n}\text{máx}(T_0-T_k,0)$.

The valuation of European call options over the daily cooling index with the α-stable model is:

$$c(\tau,C_\tau)=\exp(-i\tau)\left((C_0-S)\Phi(d,\alpha,-\beta)+\gamma f_{Y_\tau}(d,\alpha,-\beta)\right)$$
$$c(\tau,C_\tau)=\exp(-i\tau)\left((C_0-S)(1-\Phi(-d,\alpha,\beta))+\gamma f_{Y_\tau}(-d,\alpha,\beta)\right) \tag{15}$$

where $d=\dfrac{C_0-S_C}{\gamma}$ and $C_0=\sum_{k=1}^{n}\text{máx}(T_k-T_0,0)$.

Valuation of call options on temperature indices

Valuation of European call options on temperature indices is performed using 2018 data. The results of the monthly indices are presented in Table 10.

Table 10
Temperature indices 2018

2018	H_0	C_0
January	106.75	0.00
February	76.75	1.45
March	7.70	39.20
April	5.05	62.85
May	0.95	118.55
June	0.95	94.30
July	6.70	32.70
August	4.80	33.10
September	10.55	26.50
October	28.85	22.50
November	73.90	1.75
December	123.54	0.00
Total	446.49	432.90

Source: created by the authors with data from the Tacubaya central observatory

Table 10 presents the daily heating and cooling degree indices for 2018, and negative skewness is observed in the total heating degrees to cooling degrees, i.e., the annual heating degree demand is 446.49°C, and the annual cooling degree demand is 432.90°C. The heating degrees present seasonal demand where the minimum demand occurs in May and June, increasing from July to December, reaching a maximum in December, and decreasing from January to June. The cooling degrees also present seasonal demand where the minimum demand is in December and January, increasing from February onwards, reaching a maximum in May, and decreasing from June to November. The proposed settlement prices assuming capacity or need to purchase heating and cooling degrees for 2019, are presented in Table 11.

Table 11
Settlement prices for 2019

2019	S_H	S_C
January	100	1
February	70	2
March	15	35
April	8	60
May	3	110
June	1	90
July	3	30
August	8	30
September	15	25
October	35	20
November	70	2
December	110	1
Total	438.00	406.00

Source: created by the authors

Table 11 presents the settlement prices for 2019; they are a proposal based on the information of the observatory data from 1958 to 2018 and are an assumption of the capacity or need to purchase heating or cooling degrees for 2019, posited in an over-the-counter market where investors with the long position seek a hedge according to business needs to minimize the cost of capital for the demand of heating or cooling degrees based on the temperature of Mexico City. The valuation of the European call options on the heating and cooling indices is presented in Figure 5.

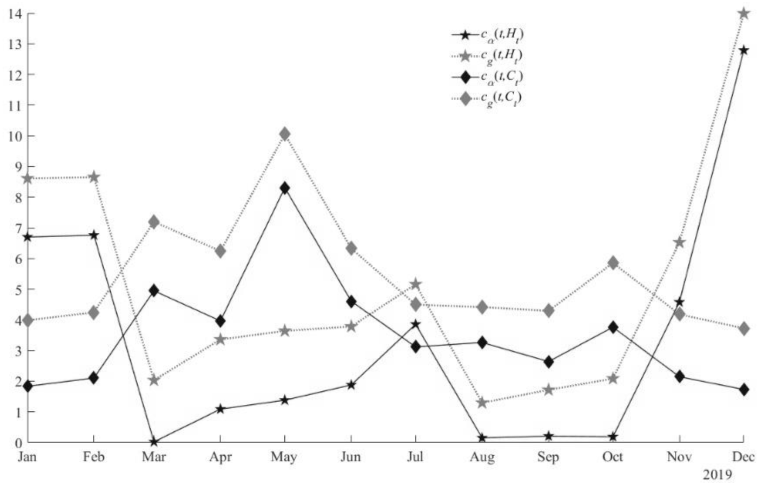

Figure 5. Valuation of European call options on indices
Source: created by the authors

Figure 5 presents the European α-stable and Gaussian call options on the valuation of monthly heating and cooling indices valued at the start of 2019. Gaussian options are more expensive than α-stable options, but the objective is not to compare the models because the α-stable model is statistically meaningful. Gaussian options are more expensive than α-stable options and annual hedges are:

$$c_\alpha\left(t,C_t\right)=42.41, c_g\left(t,C_t\right)=65.01, c_\alpha\left(t,H_t\right)=39.60, c_g\left(t,H_t\right)=60.84$$

where the annual Gaussian hedge over the cooling index (65.01) is higher than the α-stable hedge (42.41), and the annual Gaussian hedge over the warming index (60.84) is higher than the α-stable hedge (39.60). The valuation of the European call options is performed monthly, and the annual premium is calculated as the present value of the twelve hedges. Based on the stochastic mean-reversion model for 2019, the option settlement payments are calculated, and the results are presented in Table 12.

Table 12
Settlement payments on call options

2019	H_T	C_T
January	0.00	0.41
February	0.00	12.60
March	0.00	6.36
April	0.00	32.55
May	0.00	0.00
June	0.00	0.00
July	0.00	27.02
August	0.00	13.40
September	0.00	0.00
October	0.00	5.15
November	0.00	0.00
December	0.00	0.00
Total	0.00	97.49

Source: created by the authors

Table 12 presents the settlement payments. Assuming, for simplicity, that the cost per degree of heating or cooling is a monetary unit and the required heating degrees are less than the settlement degrees, then the call options on the heating index are out of the money. On the other hand, the cooling degrees from January to March, July, August, October, and December are greater than the settlement degrees, recovering the premium for the hedge; therefore, the proposal is that the reference temperatures are statistically estimated based on the location, and the needs of the product or service and the costs that these needs have for the estimation of the indices and hedges. In Mexico City, cooling degrees (449.64) are more necessary than heating degrees (267.39); therefore, the reference temperature is a parameter depending on the temperature needs of the product or service and the geolocation.

If the company takes the long position in the bond invested at the risk-free interest rate to cover the daily cooling degrees at the end of the year, $S_C \exp(-iT)$, to receive USD 406 and takes the long position in the call option on the cooling degrees, then, it saves USD 449.64 on energy consumption costs for cooling for a premium of USD 42. 41 and, if the company takes the long position in the bond invested at the risk-free interest rate to cover the heating degrees at the end of the year, $S_H \exp(-iT)$, to receive USD 438 and takes the long position in the call option on the heating degrees, then, it has the USD 438 for energy consumption costs to heat for a premium of USD 39.60, which expires without generating savings because it is out of the money.

The importance of having information for estimating indices to value hedges with derivative products is evident.

Conclusions

Temperatures show seasonal behavior, a positive and significant trend, with the coldest periods occurring between October and March and the warmest periods occurring between March and June. The analyzed temperatures present leptokurtic distributions with negative skewness; therefore, α-stable distributions are relevant to model the stochastic process of reversion to the mean for the valuation of climate options on temperature indices. The average temperature increases during the years analyzed are 2.04°C and significant.

The deterministic mean-reverting model is significant and indicates that the amplitude of the periodic variation function is 2.35°C, i.e., the difference between average temperatures is 4.7°C, so the deterministic mean-reversion model captures seasonal behavior and positive trends more adequately than the simple linear regression model; therefore, the deterministic mean-reversion model is significant for modeling temperatures but is insufficient for valuing call options on temperature indices and valuing the hedge for the company.

The objective of evaluating climate options on temperature indices is achieved with the proposal to apply a stochastic differential equation using a stochastic α-stable process and a solution to simulate temperatures. The monthly α-stable parameters are estimated and are significant, concluding with AD and CvM goodness of fit tests that the valuation of options on temperature indices in incomplete markets is relevant with α-stable distributions.

The estimator for the mean-reversion parameter is proposed as a function of the scaling parameter that enables simulated random variables to forecast 2019 daily temperatures; the final condition for the price of options on temperature indices is indicated, and a model for the valuation of European call options on temperature indices is proposed with α-stable distributions. Then, the call options on the indices are valued, and it is concluded that the α-stable options are relevant and statistically significant and in all cases have prices lower than or equal to the Gaussian options. The temperature index options represent an opportunity to minimize the cost derived from energy consumption to increase or decrease the temperature and a benefit for companies and consumers. The main contributions of this work are applying a stochastic differential equation with α-stable processes justifying its relevance, estimating the mean reversion parameters, and solving the stochastic differential equation to value options on temperature indices with a closed equation that is an application of statistically significant α-stable processes. Furthermore, the solution of the stochastic differential equation makes it possible to apply the Monte Carlo simulation to value options that do not have closed solutions applying α-stable processes and considering statistically significant seasonal, cyclical, warming, and mean reversion effects.

The work highlights the importance of having temperature information through the Internet and climatic indices depending on the location because temperature is a cost factor for businesses. It is possible to minimize economic losses due to energy consumption by hedging for the demand for the necessary heating or cooling rates.

For future research, it is possible to compare the model with the prices of options on the degrees of warming or cooling for the indices traded in the Mexican organized or over-the-counter markets.

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