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Estimation of credit risk in infrastructure projects using structural models

Estimación del riesgo de crédito en proyectos de infraestructura mediante modelos estructurales

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Abstract

This paper aims to implement a credit risk model in infrastructure investment projects, where the probability of default is estimated considering the cash flow available for debt service, which determines the debt service cover ratio. For that, a structural model developed for illiquid assets is used, such as an extension of the credit risk models such as Merton (1974) and KMV of Moody's, through which the components of the probability of default, exposure, recovery rate and expected loss are analyzed. The main innovation of this approach is due to the incorporation of a dynamic of the debt service cover ratio, which is modelled stochastically following the same assumptions of the option pricing theory. In addition, this model is complemented with the Monte Carlo simulation technique, under which some main parameters are estimated, as well as the expected loss and the credit Value-at-Risk (VaR).

JEL Code: C14, G13, G21
Keywords: credit risk; probability of default; stochastic process

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Resumen

Este trabajo tiene como objetivo implementar un modelo de riesgo de crédito en proyectos de infraestructura, donde se estima la probabilidad de incumplimiento teniendo en cuenta el flujo de efectivo disponible para el servicio de deuda y su ratio de cobertura. Para ello, se utiliza un modelo estructural de riesgo de crédito desarrollado para activos líquidos, como una extensión de los modelos de Merton (1974) y KMV de Moody's, mediante el cual se analizan los componentes de probabilidad de incumplimiento, exposición, tasa de recuperación y pérdida esperada. La principal innovación de este enfoque se debe a la incorporación de una dinámica propia de la ratio de cobertura del servicio de deuda, el cual se modela estocásticamente siguiendo los mismos supuestos de la teoría de valoración de opciones financieras. Además, este modelo se complementa con la técnica de simulación de Monte Carlo, bajo la cual se estiman algunos parámetros esenciales, así como la pérdida esperada y el valor en riesgo (VaR) crediticio.

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Introduction

Project Finance involves the design of a complex financing structure that links private investors together with the aim of financing, building, and managing complex infrastructures, generally of a public nature (Grimsey & Lewis, 2002). The main objective of this scheme has been to increase government efficiency through the generation of incentives[1] and the allocation and transfer of risks[2] (Yescombe, 2002; Gatti, 2008). Under this scheme, a special purpose vehicle (SPV) is created on an ad hoc basis independent of the sponsors and with limited resources from them.

In addition, according to Gatti (2008), project finance has distinctive characteristics compared to traditional corporate finance, such as: i) it is a structured form of off-balance sheet financing; ii) the SPV requires a level of specificity that determines its purpose and associative nature; iii) sponsors' resources are limited or zero, which represents a high level of leverage with long terms to recover the investment; and iv) the financing incorporates significant and extensive control rights for lenders, as well as covenants and restrictive clauses.

[1]The limitations of public resources for infrastructure investment, the restrictions on their borrowing capacity, mainly in developing countries, together with the need to improve and expand the provision of public goods and services, have led governments to turn to private investors to participate in long-term contractual arrangements known as public-private partnership (PPP) agreements and private finance initiatives (PFI).
[2]The allocation of project risks among the various financing participants is crucial in determining the success of the project. Similarly, project financing is based on the establishment of a complex network of contracts in which different parties involved with different roles are identified and where the contracts entered into by the SPV not only function as a guarantee to access financing, but also allow the transfer of risks to the counterparty that can best manage them.

Thus, all economic consequences of the project are directly attributed to the SPV. Therefore, debt repayment depends primarily on the project's ability to generate cash flow[3], determining its creditworthiness. Cash flows are considered the main source of resources to cover the debt obligations incurred, i.e., once the project has covered its operating requirements (costs, maintenance, expenses, taxes, among others) from the revenues generated. Thus, cash should be sufficient to pay the debt service (principal plus interest). Added to this is the absence of a project credit history to serve as a basis for a financing decision, as Cappon et al. (2018) indicated.

Because of these characteristics, the credit analysis must determine whether the investment vehicle can pay the debt contracted. The problem lies in the future uncertainty about the success of the project, given the different risks that are present in its life cycle phases, for example, a delay in the completion of construction due to natural or social events, a reduction in its demand or revenues, or an increase in operating costs, among others (Cartea & Figueroa, 2005; Boussabaine, 2014). In addition, any breach of the contractors' contractual obligations may disrupt the proper functioning of the project. This uncertainty translates into the probability of defaulting on the financing agreement[4], that is, a default on debt service payments, either partially or in full.

Consequently, the granting of debt in infrastructure projects, mainly private initiative, has incorporated (significant and extensive) control rights for lenders (Borgonovo & Gatti, 2013; Blanc-Brude, Hasan, & Ismail, 2014), as well as covenants and restrictive clauses (Gatti, 2008; Blanc-Brude, Hasan and Whittaker, 2016), while prohibiting new debt issuance from repaying existing debt. Therefore, credit analysis in infrastructure projects becomes much more complex (Jobst, 2018; Wang et al., 2019) compared to the corporate sector.

As a result, a rigorous credit risk analysis needs to reflect these characteristics. Nevertheless, traditional credit assessment models: i) reduced form models, ii) rating-based models, and iii) structural models, cannot be directly applied or present strong limitations for this type of financing, as found in previous studies by Klompjan and Wouters (2002), Gatti et al. (2007), Kong et al. (2008), Dong et al. (2012), Karminsky and Morgunov (2016), and Wang et al. (2019), among others. In addition, Klompjan and Wouters (2002) and Karminsky and Morgunov (2016) agreed that there are limitations associated with the limited availability of (historical) default data as well as difficulties in accessing this information, along with the diversity in the purpose of each project and its risks, which makes it difficult to determine the explanatory variables.

[3]According to Boussabaine (2014), a structural characteristic of the SPV is its limited discretion in project management, focusing as it does all its efforts on cash flow generation, with which it seeks to cover the obligations incurred and remunerate all parties involved.
[4]In this regard, Jobst (2018) finds that infrastructure projects achieve a recovery rate of close to 80% (on average).

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Besides these limitations, it is found that the above models do not allow the incorporation of not only the effects of debt covenants but also the dynamic nature of the project's payment capacity, as indicated by Blanc-Brude et al. (2016). Although this last drawback is overcome in the field of structural models, as shown by Freydefornt (2001) and Aragones, Blanco, and Iniesta (2009), who implemented models for estimating credit risk in infrastructure projects based on Merton's (1974) and Moody's KMV models, limitations remain. In response, Blanc-Brude and Hasan (2016) proposed extending Moody's KMV model to estimate the probability of default in infrastructure projects. Unlike previous applications, they estimate the probability of default based on a stochastic treatment of the project's payment profile dynamics, which is determined through cash on hand. This model was extended by Blanc-Brude et al. (2016) to estimate the conditional probability of transition between states (from a risky to a safe state and vice versa) by integrating Bayesian inference techniques.

Similarly, Wang et al. (2019) proposed an adjusted CreditMetrics model with Monte Carlo simulation to estimate credit risk based on information provided by project cash flows. Using a qualitative analysis supported by Standard & Poor's "recovery scale," they overcame the limitations associated with the lack of information and estimated the expected loss and its components. Nonetheless, this application presents skews by incorporating subjective elements in the estimation, which the authors themselves highlight.

Based on these developments, this paper aims to implement a credit risk estimation model, where the probability of default in infrastructure projects is estimated by incorporating the dynamics of the payment profile using the available cash flow and the debt service coverage ratio. For this purpose, a structural credit risk model developed for illiquid assets is adopted following the work of Blanc-Brude and Hasan (2016) and Blanc-Brude et al. (2016). The exposure and potential loss for a hypothetical toll road concession project are also analyzed. This methodology represents an extension of the traditional credit risk models of Merton (1974) and Moody's KMV, and the main innovation of this approach is due to the incorporation of proprietary dynamics of the debt service coverage ratio (DSCR), which is stochastically modeled following the same assumptions of the financial option pricing theory. In addition, this model is complemented by the Monte Carlo simulation technique, under which some essential parameters are estimated, such as expected loss and value at risk (VaR).

Structural credit risk models

Since the seminal works of Black and Scholes (1973) and Merton (1973), financial option pricing models have been adapted to address corporate problems, including credit risk assessment. These developments are initially found in the work of Merton (1974) and Merton (1977), as well as in extensions by Black and

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Cox (1976) and Ingersoll (1977). This paper identifies all these developments as the "Merton model." This field comprises the beginning of the credit risk estimation approach based on the structural models[5] presented below.

Merton model

Merton (1974) extended the Black-Scholes formula to contingent claims analysis (CCA) to treat corporate problems, where he assumes that a company's debt can be considered a claim on its assets, with an exercise price equal to its nominal value and a given maturity date. In this way, he proposes a relation between the capital structure and the company's ability to service its debt. For this purpose, it is assumed that the evolution of the market value of the company's assets (V_A) follows a stochastic process of the geometric Brownian motion type (mBg) given by equation 1.

$$dV_A = \mu_A V_A dt + \sigma_A V_A dW_t \tag{1}$$

Where μ_A represents the asset value drift rate, σ_A its volatility and $W_{t \in [0,T]}$ is a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$ with a filtration $(\mathcal{F}_t)_{t \in [0,T]}$.

The value of the firm's assets is determined by $V_A = V_E + V_D$ at an initial time t=0, where V_E is the market value of the firm's shares, and V_D is the value of its debt. In addition, it is assumed that D_V is represented by the issue of a zero-coupon bond maturing in T and with value $B(t, T)$. Then, if $V_A < V_D$ then the company defaults on its debt, in this case $V_E = 0$, while if $V_A > V_D$, the company pays its debt in T and, therefore, $V_E = V_A - V_D$. This logic can be represented as a function of the form:

$$V_E = \max(V_A - V_D, 0) \tag{2}$$

Thus, V_E is assimilated to a European-style call option with a strike price equal to D_V. This indicates that if the value of the assets is insufficient to meet the debt, then the shareholders, who hold the call option, will not exercise their right and leave the company to their creditors.

Now, equation 1 can be represented in logarithmic terms. Therefore:

[5]Unlike reduced-form models and those based on ratings, structural models assume that investors have complete information about the market and therefore have knowledge of the value of the companies' assets and debt.

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$$\ln V_A = \ln V_0 + \left(\mu - \frac{1}{2}\sigma_A^2\right)t + \sigma_A Z\sqrt{T} \tag{3}$$

Where $Z \sim N(0.1)$ and, the probability of default of the company ($P(t,T)$) between t and T, is given by

$$P(t,T) = P[\,V_A \le V_D] = P[\,\ln V_A \le \ln V_D] \tag{4}$$

Then, in a risk-neutral world, the value of V_E is determined by the formula

$$V_E = V_A\,N(d_1) - V_D\,e^{-rT}N(d_2) \tag{5}$$

Where r represents the risk-free interest rate and $N\,(\cdot)$ indicates the cumulative normal distribution function of the parameters d_1 and d_2, as shown in Equations 6a and 6b.

$$d_1 = \frac{ln\left(\frac{V_A}{V_D}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} \tag{6a}$$

$$d_2 = \frac{ln\left(\frac{V_A}{V_D}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}} = d_1 - \sigma_A\sqrt{T} \tag{6b}$$

Finally, the probability of default (risk-neutral) is the probability that V_A at T is below D_V. Then

$$P\,[V_A \le V_D] = N(-d_2) \tag{7}$$

An important limitation of the Merton model is that the parameters of the process are determined by equation 1, i.e., μ_A and σ_A, are not directly observable, which makes its application difficult.

Moody's KMV model

Moody's KMV model was initially developed by Vasicek (1984) and later completed by McQuown (1993) and Kealhofer (1993) as an extension of the Merton model to estimate a probability of default based on the notion of distance to default (DD). Unlike the Merton model, the KMV model assumes that the

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company defaults when the value of its assets falls below a threshold defined by the value of its debt, which determines the point of default.

Figure 1 shows its application based on the representation of a possible trajectory of the market value of the assets. There, it is observed that the company has no way to pay the debt if the value of assets falls below the default point. Therefore, the probability of default is the probability that the asset's value will fall below this point. This probability is represented in the shaded area of the distribution function below the default point.

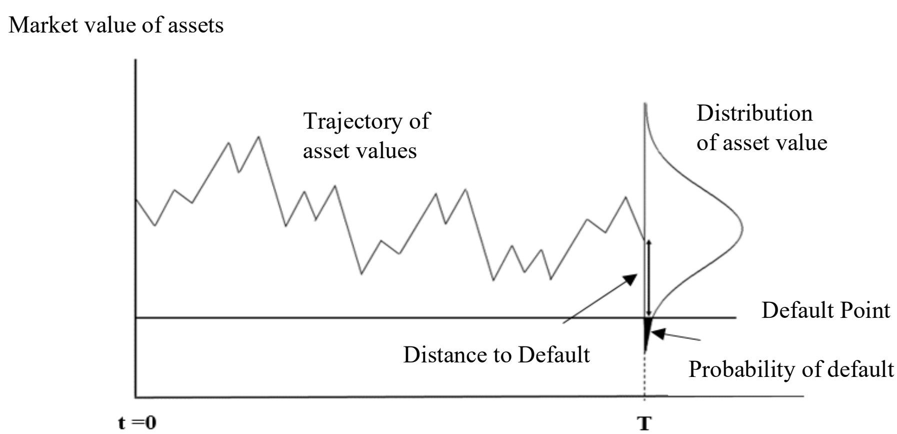

Figure 1. Representation of default based on distance to default

Source: adapted from Kealhofer (2003).

As a result, the model estimates the distance to default as the number of standard deviations by which the asset value exceeds the default point. Then, the distance to default (DD) is computed as

$$DD = \frac{V_A - DP}{\sigma_A \, V_A} \tag{8a}$$

Where V_A represents the value of assets and σ_A their volatility. For a limited liability company, the equity, which is determined by the market value of the shares (E_V), has the residual claim on the assets after all other obligations have been fulfilled. Consequently, a call option on assets with a strike price equal to the debt has the same properties as those indicated in Merton's model. Thus, taking the same assumptions about V_A, V_E and V_D, given by equations 1 and 5, DD could be estimated using equation 6b. Then

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$$DD = \frac{ln\left(\frac{V_A}{V_D}\right) + \left(r - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

(8b)

Nevertheless, since σ_A is not observable, DD is approximated by equation 8b, while the probability of default (risk-neutral) at a maturity T is determined by

$$P\,[V_A \leq V_D] = N(-DD)$$

(9)

As a result, this model establishes that DD is sufficient to estimate the probability of default, where the numerator of equation 8b determines the level of financial leverage associated with the company's level of financial risk. At the same time, the denominator reflects its level of market risk.

Credit risk estimation model for infrastructure projects

The estimation of the probability of default, together with the other components required for the quantification of credit risk (expected loss) in the corporate sector, has been widely studied in financial theory, where credit evaluation is performed based on the information provided by balance sheets and financial statements, as well as their historical performance (Gatti et al., 2007). In this context, structural models present advantages for their estimation; however, in infrastructure projects, the development of proprietary models has been scarce, and the above models present limitations. In this regard, the Basel Committee in the framework of the Basel II Accord (Basel, 2004) highlighted the limitations of using conventional credit risk models[6] in the structured financing of infrastructure projects. Based on the distinction between corporate finance and structured project finance, the Committee recommended the implementation of different methodologies to determine the expected loss of credit risk (EL), considering the three components:

$$EL = DP\ x\ LGD\ x\ EA$$

(10)

Where PD is the probability of default, LGD is the loss given default, and EAD is the exposure at default.

[6]This distinction is made to refer to the Merton and Moody's KMV models.

As indicated above, in project finance, cash flow is the determining factor in the value of the assets and therefore determines the debt repayment capacity. Initially, this value is the only benchmark[7] considered in the project's financial structuring to determine the debt's size. Therefore, credit risk is associated with the future uncertainty of the project's cash flows, and its dynamics determine the payment capacity and the probability of default. Thus, the assessment of credit risk is determined by the possibility that, at any given time, cash flows will be insufficient to service debt.

Recent work has shown that structural models can be used to estimate credit risk in projects (see Blanc-Brude et al., 2014; Blanc-Brude & Hasan, 2016). Nevertheless, these models merit special treatment with some adjustments. For example, the incorporation of control rights for lenders (Borgonovo & Gatti, 2013) and debt covenants significantly affect the estimation of expected loss and its components, thus requiring a redefinition of default. Similarly, the model specification regarding cash flow allows the use of observable information to obtain the model parameters. For example, Blanc-Brude and Hasan (2016) showed that understanding the dynamics of the debt service coverage ratio ($DSCR$), together with the amortization profile and available securities[8], which are observable, is sufficient to implement a structural model.

The first key step in estimating credit risk is the modeling of cash flows for each point in time t, where default occurs in those scenarios when cash flows are insufficient to pay debt service (principal and interest). Thus, determining the project's ability to pay based on DSCR is essential for its application. $DSCR$ measures the amount of cash available to make debt service payments (CFADS) at time t, and is estimated as

$$DSCR_t = {}^{CFADS_t}/_{DS_t} \tag{11}$$

Where DS_t represents debt service at t. The higher the DSCR, the more cash the project will have to meet its debt obligations. DSCR dynamics facilitate the implementation of a structural credit risk model since DSCR levels determine default thresholds. According to the authors, the DSCR provides an explicit definition of the point of default as:

i. "hard" default, determined by $DSCR = 1$;
ii. "technical" default, determined by $DSCR = 1, x$.

[7]According to Gatti (2008), the project generally possesses assets that define a specific character determined by its contractual design
[8]One form of collateral for lenders is that which is available at the "tail" of the life of the debt. According to, Blanc-Brude and Hasan (2016), the presence of these distinctive features makes structural credit risk models a natural choice.

Thus, knowledge of the $DSCR$ dynamics is enough to estimate the distance to default (DD). Similarly, $DSCR$ dynamics can also be combined with future debt service to calculate the expected value and volatility of future cash flow. Despite their estimation of both DD and the probability of default, the other components of the EL have limitations that should be noted. Generally, lenders, in their effort to reduce their risk exposure, require the SPV to create cash reserve accounts to cover part of the future debt service, which increases their recovery rate and reduces the total exposure in the event of a default event. Therefore, the estimation of these components requires a stepwise analysis, as will be discussed below.

Dynamics of DSCR and determination of the probability of default (non-compliance)

Since the behavior of the $DSCR$ is strongly related to the project's credit risk, knowledge of the $DSCR$ distribution is sufficient for its estimation. If the stochastic dynamics of the DSCR follows a log-normal process, then

$$d\,DSCR_t / DSCR_t = \mu\, dt + \sigma\, dW_t \tag{12}$$

Where μ and σ represent the DSCR drift rate and corresponding volatility, which are known and constant, while $W_{t \in [0,T]}$ is a standard Wiener process. Now, since the point of (technical) default is determined by

$$DSCR_t \equiv CFADS_t / DS_t < 1. \tag{13}$$

Where CFADS is the cash flow available for debt service, then analogous to Moody's KMV model, the distance to default at each t is

$$DD_t = \frac{CFADS_t - DS_t}{\sigma_{CFADS}\, CFADS_t} \tag{14}$$

Where σ_{CFADS} is the volatility of CFADS. In order to avoid any problems due to the scale dependency of CFADS, the cash flow is re-expressed in terms of DSCR

$$CFADS_t = DSCR_t \times DS_t \tag{15}$$

Using this CFADS definition, the distance to default can be expressed as

$$DD_t = \frac{1}{\sigma_{CFADS}}\left(1 - \frac{1}{DSCR_t}\right) \tag{16}$$

Similarly, Equation 16 can be rewritten as a function of DSCR alone by expressing CFADS volatility as a function of DSCR volatility (σ_{DSCR})

$$DD_t = \frac{1}{\sigma_{DSCR}} \frac{DS_{t-1}}{DS_t}\left(1 - \frac{1}{DSCR_t}\right) \tag{17}$$

Where $\sigma_{CFADS} = \left(\frac{DS_{t-1}}{DS_t}\right)\sigma_{DSCR}$[9]. Similarly, the probability of default (Pt,T) is determined by DDt:

$$P(t,T) = N(-DD_t) \tag{18}$$

Nonetheless, this measure incorporates the risk preferences of investors. Since Equation 12 shows the dynamics of the DSCR in terms of the drift rate (μ), the probability estimation requires a risk-neutral adjustment, where the risk premium (Sharpe coefficient λ) is introduced, which is estimated for a horizon T: $\lambda = \frac{\mu - r}{\sigma}\sqrt{T}$[10]. Thus,:

$$Q(t,T) = N(N^{-1}[P(t,T) + \lambda) \tag{19}$$

Where $Q(t,T)$ is the risk-neutral default probability. Similarly, estimating the probability of default can also be performed in a Monte Carlo simulation context, where possible trajectories of the $DSCR_t$ and $CFADS_t$ are generated, seeking to reflect the largest number of possible trajectories. Therefore, by modeling the cash flows, it is possible to reflect whether a default event may occur during the project's life

The advantage of this simulation approach lies in its practical ability to incorporate the various sources of project risk associated with operational and market uncertainties through probabilistic

[9]It should be noted that if, given the credit conditions, a fixed installment amortization schedule is adopted, then: $\sigma_{CFADS} = \sigma_{DSCR}$, given that $DS_{t-1} = DS_t$, for the entire period.
[10]This risk-neutral adjustment aims to incorporate investors' risk preferences into the *CFADS* modeling. This indicates that, if the cash flows reflect the risky nature of the investment, they should be discounted at the risk-free rate in the valuation model.

assumptions that reflect their very nature[11] and correlations. This way, identifying project risk factors and their modeling can have practical advantages. The implementation of the Monte Carlo simulation model in stages is detailed below.

The first stage of the Monte Carlo model implementation, as shown in Figure 1, involves incorporating uncertainties into the financial model (the cash flow) by assigning probabilistic assumptions on the different sources of uncertainty in the project. This process results in a probability distribution function that characterizes the present value of the project's cash flows. The rate of change in the present value of cash flows comprises the main input for estimating the volatility of the $DSCR$ (σ_{DSCR}).

The second stage of the Monte Carlo model implementation (see Figure 3) comprises the simulation of $DSCR_t$ trajectories, where it is assumed that it follows a stochastic process of the mBg type. Then, for each time t the probability of default is estimated, i.e., falling below a threshold defined in $1,x$. The default point is defined based on the covenants imposed in the debt contract.

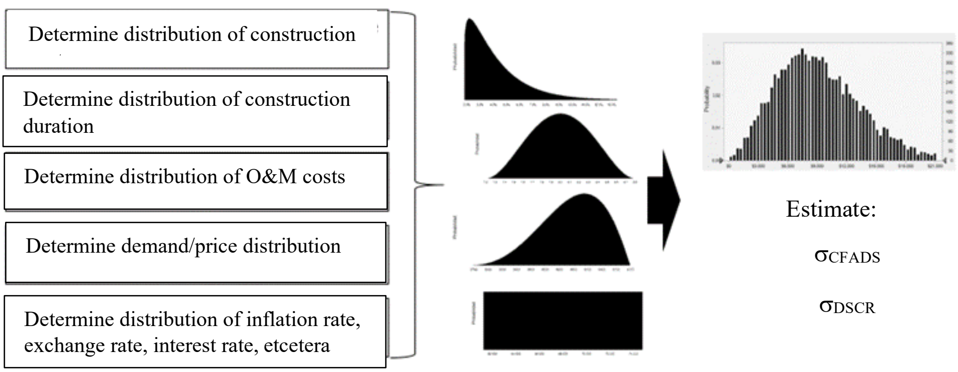

Figure 2. Stage 1: Modeling uncertainties and volatility estimation

Source: created by the author

[11]The advantage of adopting different types of probability functions is that the model parameters can be obtained from empirical evidence.

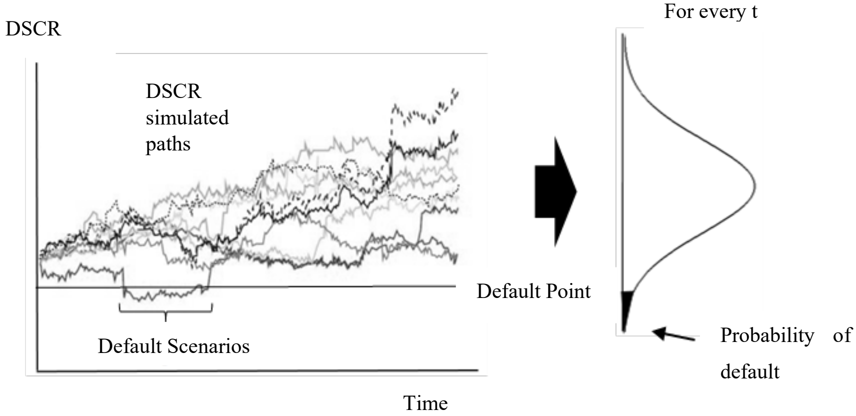

Figure 3. Stage 2: Simulation of DSCR trajectories and estimation of the probability of default
Source: created by the author

In addition, this type of application of the Monte Carlo simulation technique is used by Gatti et al. (2007) to estimate the project's value at risk (VaR). The reader should keep in mind that the modeling of risk factors represents a critical issue, where one's judgments and criteria greatly affect the risk assessment and, therefore, the estimation of the probability of default.

Restructuring of the financing agreement and analysis of default

So far, credit risk has been analyzed in a context where the probability of default is determined by the project's ability to pay. Nevertheless, the above analysis should be extended to incorporate the effect of restructuring the debt contract in the event of a default. This is because lenders may agree to a restructuring, rather than allowing the project to default on the debt given the adverse consequences such as loss of credit quality and increased financing costs.

According to Gatti (2008), the debt contract may incorporate clauses that allow lenders to take control measures to prevent the project's repayment capacity from being affected. Therefore, in the event of default, lenders will be willing to take corrective measures to keep the project in normal operation[12], measures that may even generate a loss for them, such as when the restructuring extends amortization terms, reduces interest payments, or, on the contrary, accelerates its recovery rate. For this reason, credit

[12]These corrective measures commonly allow them to renegotiate the current terms of the debt, changing the amortization terms, interest payments, and the incorporation of subordinated debt.

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risk components such as LGD and EAD may change in a default scenario, and estimation models must incorporate these adjustments.

Although their application can be complex, they can be incorporated from: i) Black-Cox decomposition from the structural model; and ii) Monte Carlo simulation in stages. These proposals are detailed below.

Black-Cox decomposition

The Black-Cox decomposition, proposed by Black and Cox (1976), comprises a method used to value corporate debt in restructuring or refinancing scenarios when the value of the company's assets reaches a limit (lower or upper). Thus, the model proposed by Blanc-Brude and Hasan (2016) should require additional adjustment, given that it only considers the dynamics provided by cash flow. This adjustment is presented based on a decomposition incorporating the four payment functions below.

1. $P(T_D, CFADS_{T_D})$: final payment at maturity of the debt, where T_D represents the maturity of the debt.

2. $\underline{P}(\tau, \underline{CFADS_\tau})$: value of assets if CFADS reaches the lower limit, namely a default, at time τ, leading to a state of restructuring.

3. $\overline{P}(\tau, \overline{CFADS_\tau})$: value of assets if CFADS reaches the upper limit in time τ and corresponds to a refinancing state because it allows the debt repayment to be accelerated, together with a reduction in its costs.

4. $p'(t, CFADS_t)$: debt payment made before maturity or restructuring.

Thus, the total value of the assets corresponds to the present (expected) value of the sum of the four payment functions under the risk-neutral probability measure. If $h(V_t, t)$ is defined as the value of assets at time t, $K(\cdot)$ denotes the interval $\underline{CFADS_\tau}(\cdot), \overline{CFADS_\tau}(\cdot)$, and V_t is the value of all future cash flows, then for each payment function:

$$h_1(V_t, t) = E\left[e^{-r_{T_D,t}(T_D - t)} P(T_D, CFADS_{T_D})\right] = e^{-r_{T_D,t}(T_D - t)} \int_{k(T)} (T_D, CFADS_{T_D}) dF^* \tag{20}$$

$$h_2(V_t, t) = \int_t^T e^{-r_{\underline{CFADS},t}(T_{\underline{CFADS}} - t)} \times \underline{P}\left(CFADS_{T_{\overline{CFADS}}}, T_{\overline{CFADS}}\right) dF^*_{T_{\overline{CFADS}}} \tag{21}$$

$$h_3(V_t, t) = \int_t^T e^{-r_{\overline{CFADS},t}(T_{\overline{CFADS}}-t)} \times \overline{P}\left(CFADS_{T_{\overline{CFADS}}}, T_{\overline{CFADS}}\right) dF^*_{T_{\overline{CFADS}}}$$

(22)

$$h_4(V_t, t) = \int_t^{T_D} e^{-r_{s,t}(s-t)} \times \left[\int\int_{k(T)} p'(CFADS_s, s) dF^*(CFADS_s, s)\right] ds$$

(23)

Where dF^* is the probability that $CFADS$ falls below the threshold during T_D, $F^*_{T_{\underline{CFADS}}}$ defines the density function when $CFADS$ reaches the default limit, while $F^*_{T_{\overline{CFADS}}}$ defines the density function that reaches the upper limit (refinancing).

Thus, the total value of the assets $V^s(V_t, t)$ will be determined by

$$V^s(V_t, t) = \sum_{i=1}^{i=4} h_i(V_t, t)$$

(24)

Where $h_i(V_t, t)$ does the i-th payment function determine the value.

Step-by-step simulation based on the Monte Carlo simulation technique

Incorporating restructuring events into the simulation requires first determining, as in the case of a default, the triggers that affect the LGD and EAD components. Regarding the LGD, it is known that the recovery rate depends on future $CFADS$ and, therefore, on the value of the project in case of default. Following Gatti (2008), a third level (stage 3) should be added to the process indicated in the previous section. Once the Monte Carlo model results are obtained (stages 1 and 2), a set of simulations must be run for each default scenario to obtain the distribution of LGD values. In addition, this exercise can provide additional information relevant to the project risk analysis, such as the VaR defined at a confidence level (e.g., 99%) and a given time horizon, or for each year of the debt's life.

Application in a road concession project

The structural credit risk model presented will be implemented in a hypothetical toll road concession project in Colombia, structured under a BOMT (build-operate-maintenance-transfer) scheme. The objective of this application is to guide the reader in the step-by-step implementation of the proposed model, although a complete approach will not be given.

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Project information and assumptions

The project comprises a 120 km toll road concession for 20 years, of which the first three correspond to the financing and construction phase and the remaining 17 to the operation and maintenance phase, a period in which the SPV will oversee the management of 2 toll booths. The total estimated investment for the project amounts to $716.75 billion pesos (COP), of which private investors will contribute 40%, while the remaining 60% will be obtained from local banks through a syndicated loan. This syndicated loan assumes a 12-year amortization period with a 3-year grace period (construction phase). In addition, the amortization assumes a fixed installment method (annuity) with an interest rate of 8% and a minimum coverage ratio (DSCR) of 1.2 is considered as the main covenant of the financing. Table 1 summarizes this information and the other assumptions of the project's financial model.

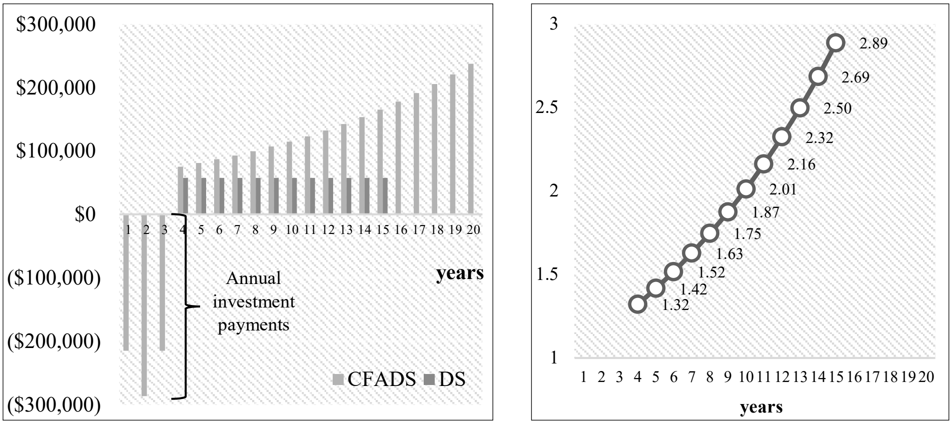
Based on this information, the CFADS and DSCR are estimated for each project year, considering that the amount of debt service under the indicated conditions is $57 065 million (COP) for the amortization period (years 4 to 15)[13]. Figures 4a and 4b present these results.

Table 1
Project assumptions

Duration	20 years
Currency	Peso COP
Expected annual inflation	3%
Total investment (in millions)	$716.750
Operating costs (in millions)	$5.700
Toll rate (average)	$19.340
Average daily traffic (ADT)	7.275
ADT annual growth	4.5%
Income tax	34%
Equity (%)	40%
Debt (%)	60%
Interest rate on debt	8%
Minimum DSCR	1.2
Cost of equity	10%
Risk-free rate	4%

Source: created by the author

[13]It should be noted that in this type of financing it is common to find the sculpted debt method for the amortization of the debt. Although this method offers distinct advantages for implementing a multistage model in Monte Carlo simulation, it will not be used in this application.



(a) Projection of CFADS and DS (b) Debt Service Cover Ratio (DSCR)

Figure 4. Estimation of the distance to default and probability of default

Source: created by the author

With these results, it is found that the project meets the debt repayment capacity by finding a minimum DSCR of 1.32 for the amortization period. Nevertheless, this result is based on a deterministic representation of the financial model and does not fully reflect the risk assumed. Therefore, this analysis should be supplemented.

Estimation of the distance to default and probability of default

The first step in estimating the probability of default corresponds to estimating the volatility of CFADS (σ_{CFADS}) or DSCR (σ_{DSCR}). It should be noted that, regardless of which one is used, under the assumed fixed installment amortization schedule, it follows that: $\sigma_{DSCR} = \sigma_{CFADS}$. Given the difficulty in estimating this parameter, it is proposed to adopt an approach like that used in real options theory[14]. For this purpose, the method developed by Brandão, Dyer, and Hahn (2012) is used. Under this model, the volatility of the present value of CFADS is obtained using the Monte Carlo simulation technique[15].

[14]It should be noted that this is not the first time that this approach has been used to address problems specific to the financing of infrastructure projects. The theory of real options has been incorporated for risk analysis and valuation, where it is assumed that the value of the project follows a diffusion process of the mBg type.

[15]In addition, by using this simulation approach, two considerable advantages are identified: i) by incorporating the different sources of risk (both operational and market) through probabilistic assumptions, their aggregate effect on the project can be estimated in a single measure; ii) these probabilistic assumptions determine the base simulation scenario, on which the LGD component or recovery rate in default scenarios can be estimated, which represents a necessary input for the estimation of the expected loss.

Table 2 presents the probabilistic assumptions used for modeling the different sources of project risk and estimating the volatility parameter.

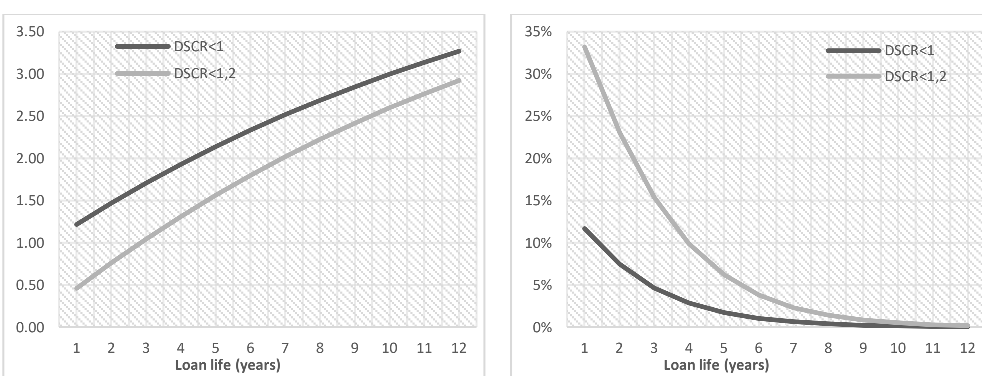
Table 2
Probabilistic assumptions of the financial model

Variable	Probability function	Parameters Mean	Standard deviation
ADT	Log-normal	6.100	540
Operating costs (in millions)	Log-normal	\$5.700	\$500
ADT annual growth	Normal	2.0%	0.2%
Maintenance	Normal	4.5%	0.45%
Inflation	Normal	3.0%	0.3%

Source: created by the author

These probabilistic assumptions on the identified variables are incorporated so as to be consistent with the diffusion process assumed in the DSCR, as indicated in equation 12. The simulation exercise employs 100.000 iterations using Crystall Ball. This results in a volatility of 18%. Once the volatility is obtained, the risk premium (λ) is estimated, which with a risk-free rate of 4% is 0.0278. As indicated by Gatti (2008), one advantage of this approach is the representation of the main sources of project risk.

The next step is to estimate the distance to default (DD) for each year of the amortization period. Likewise, the non-compliance point is defined in its two versions (hard and technical) corresponding to $DSCR_t < 1$ and $DSCR_t < 1.2$, respectively. Figures 5a and 5b present the estimates of the distance to default and the probability of default.



(a) Distance to default (b) Probability of default

Figure 5. Estimation of the distance to default and probability of default

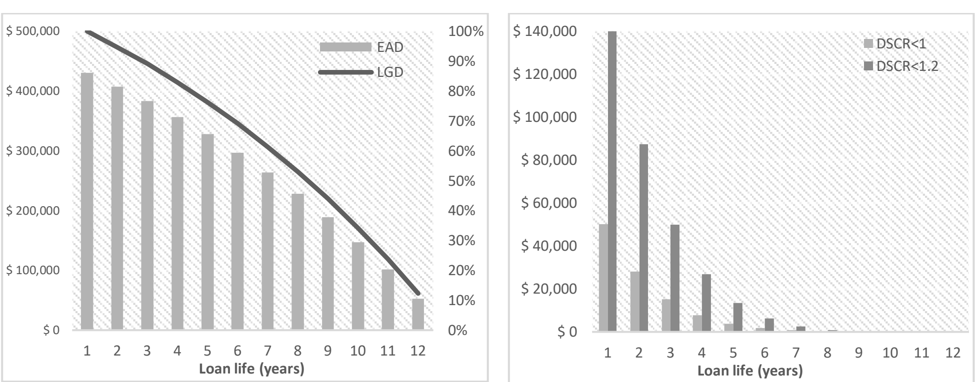
Source: created by the author

In its first year, the project reaches the lowest DSCR of the entire period (1.32). Therefore, the probability of default in this year will be the highest. As its ability to pay improves, the relation between CFADS and debt service for each t increases, and the probability decreases rapidly, which is identified for the two points of default.

Estimate of EL and credit VaR

EL takes a similar approach to the probability of default throughout the amortization period, given its direct relation with the latter and how EAD and LGD evolve, as shown in Figures 6a and 6b, differentiating the estimates for technical and hard default. The fact that this probability drops to near zero at the end of the period indicates that EL drops to zero. Specifically, from year 9 of the life of the debt it is almost zero. In other words, regardless of the other two components, the asymptotic behavior of the probability is directly reflected in EL.

The same is true for the EAD and LGD components (Figure 6a): at the end of the period, the debt has been almost completely paid off, so these two approach zero, as does the probability, and, therefore, the EL is almost zero. This dynamic is extremely useful for analyzing credit risk and the provisions of the banks that participated in its financing, as it allows a direct and simple estimation of these risks. In addition, based on this information, the present value of the total expected loss in each case is obtained: $65 550 million in the case of technical default and $221 900 million in the case of hard default. Each annual EL is discounted at the loan's interest rate (8%) to obtain these results.



(a) Estimation of EAD and LGD (secondary axis) (b) Estimation of EL

Figure 6. Estimated expected loss

Source: created by the author

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For now, only a static estimate of the EL has been made. As indicated in the previous section, additional levels or stages can be incorporated into the Monte Carlo simulation, where the EL distribution function is obtained. Once again, by using Crystal Ball, the distribution functions are obtained. Figure 7 presents the density function of the present value (PV) of EL considering the two types of default, as well as the credit VaR (at 99%), that is, the maximum probable loss of the banks that participated in financing the project.

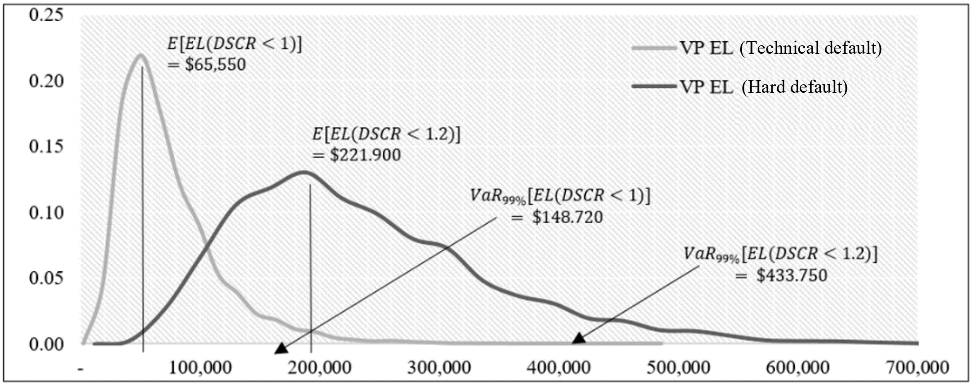


Figure 7. Probabilistic estimation of EL and credit VaR

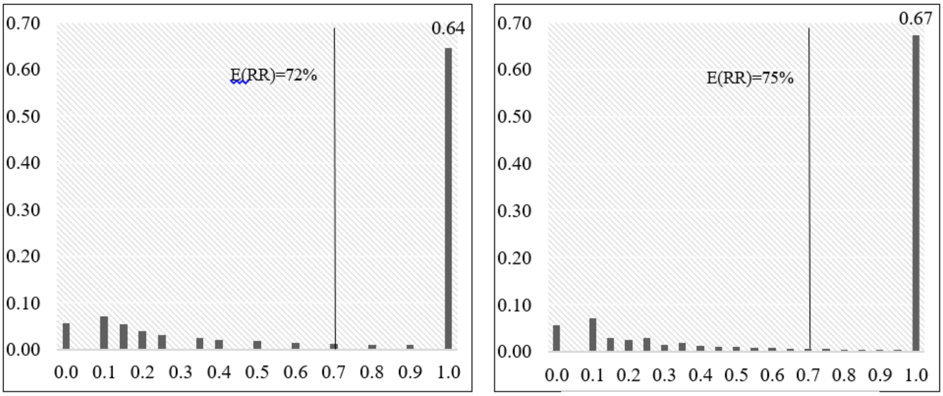
Source: created by the author

It should be noted that, for simplicity, debt reserves are not considered in the EL estimates. This is even more relevant given that the estimate depends directly on the conditions imposed by the banks, which are reflected in the covenants. The more risk that banks take on, the higher the requirements will be in debt reserve requirements.

Probabilistic analysis of RR

To conclude the analysis of the Monte Carlo simulation approach, a probabilistic analysis of the recovery rate (RR) will be performed, taking into account the dynamics of the DSCR as shown in equations 12 and 13. For this purpose, only hard default will be taken as a benchmark, i.e. $DSCR_t < 1$, where, like Blanc-Brude and Hasan (2017), it is assumed that in a hard default scenario, the SPV will be liquidated. Here the banks will be able to recover the amount of debt paid by the SPV up to date t. This analysis can be performed by taking or not taking the debt reserves accumulated up to that time, which have been ignored for now.

Figure 8 presents the simulation results. As can be observed there, the estimate of the expected recovery rate ($E[RR]$) differs somewhat, although this difference is unimportant.



(a) Without debt reserves (b) With debt reserves

Figure 8. Estimated recovery rate (RR)

Source: created by the author

The incorporation of debt reserves increases the value of $E[RR]$ from 72% to 75%, while the number of scenarios in which banks achieve full debt recovery increases from 64% to 67%. The above results are consistent with analyses by Moody's (2017) and Jobst (2018), which find that the recovery rate for the infrastructure sector globally (on average) is in the 70%-80% range.

Conclusions

This paper presented a structural model of credit risk estimation using an integrated approach that combines the dynamics of debt repayment capacity following the developments proposed by Blanc-Brude and Hasan (2016) and the Monte Carlo simulation technique. An important advantage of this approach is that it allows a direct estimation of all components of expected loss (EL), such as the probability of default (PD), exposure (EAD) and loss given default (LGD), or recovery rate (RR), considering the future uncertainty of the project's cash flow available for debt service payments and its effect on the potential loss. In addition, the proposed model can be implemented without major difficulty until a probabilistic estimation of all components is achieved.

This can be a useful tool for banks wishing to quantify credit risk in infrastructure projects. It should be noted that, given the complex nature of a default event in these projects, the treatment of credit risk is based on the thresholds determined by the DSCR, which explain the occurrence of a "hard" or "technical" default, the latter justified by the presence of covenants and significant control rights for the banks.

Similarly, there are still some limitations of the proposed model that need to be further investigated in order to present a much more robust and complete model. First, the reliability of the estimates is based on cash flow projections (CFADS) and incorporates a subjective component that is limited to the experience and knowledge of the initiative's structure and may generate some biases. Second, a proper analysis of refinancing or credit restructuring scenarios, in order to identify the optimal repayment schedule such that the recovery rate is maximized (or risk is minimized) for banks, would require the incorporation of real options theory, as suggested by Blanc-Brude, Hasan and Whittaker (2016).

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