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Lowering the cost of pensions: an alternative from the financial markets of Colombia and Mexico

Disminuir el costo de las pensiones: una alternativa desde los mercados financieros de Colombia y México

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Abstract

A relevant factor for the public finances of Latin American countries is the approximate calculation on the amount of pension funds actuarial reserves, financed through central, federal and regional entities. This article proposition is the strategy on the administration of pension fund portfolios, considering the asset pricing´s stochastic dynamic and integrating an approach on dynamic coverage these two elements allow to lower the cost on annuities applied to Colombia and Mexico. From Colombia, the strategy on pension fund administration saves approximately 28.64% of the public expense (4,583 million US dollars); for the case of Mexico, the saving is about 51.92% (22,690 million US dollars). The approach is relevant since it could liberate public resources invested in federal pension fund reserves with not only tax benefits but also social benefits mainly. The pension fund population may grow or even the resources invested could be redirected for other important people's needs.

JEL Code: C22, G22, H55
Keywords: actuarial reserves; pension coverage; financial derivatives; pension systems

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Resumen

Un elemento determinante para las finanzas públicas de los países latinoamericanos es la estimación del monto de reservas actuariales de pensiones, financiadas total o parcialmente por gobiernos centrales, federales y regionales. Se propone una estrategia de gestión de portafolios pensionales, que considera la dinámica estocástica de los precios de los activos e integra una estrategia de cobertura dinámica que permite reducir el costo de las rentas vitalicias; aplicado a Colombia y México. La estrategia representa a Colombia un ahorro promedio en el gasto público de 28.64% (4,583 millones de dólares estadounidenses aproximadamente) y para México un ahorro de 51.92% (22,690 millones de dólares estadounidenses aproximadamente). Es relevante porque permitiría liberar recursos públicos invertidos en reservas actuariales pensionales o vigencias futuras, con beneficios fiscales y sociales que ello conlleva, al dar soporte a una mayor cobertura de los sistemas de retiro o utilizarlos en otras necesidades de la población.

Código JEL: C22, G22, H55
Palabras clave: reservas actuariales; cobertura pensional; derivados financieros; sistemas pensionales

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Introduction

For insurance companies, governments, and public and private entities responsible for the payment of old age, life, or disability pensions, the calculation of the actuarial reserve necessary to cover the total payments of a life annuity is a fundamental issue. Some studies show that, in Latin American countries, the pension obtained by workers at the time of retirement is not sufficient. An example of these works is that of Gómez and Kato (2008), where they analyze how competitive pensions in Mexico compare to those granted in other Latin American countries. Their findings show that the commissions charged are higher in Mexico than in Argentina and Chile. The above implies that the monthly pension obtained by the worker at the time of retirement is lower in Mexico and therefore less competitive than in other countries. Duque, Quintero, and Gómez (2013) analyze the Colombian subsidized pension scheme from the perspective of the effectiveness of the principle of universality. They determine that said scheme is not an effective tool for its beneficiaries to access a pension. Villagómez (2014) demonstrates that, in general, individuals do not have the necessary savings for their retirement, whether they are in developed countries or not. This condition is especially true in the case of Mexico.

Meanwhile, Martínez and Venegas (2014) analyze the market risk of the mutual funds Basic Siefore 1 (SB1) and Basic Siefore 2 (SB2). An index of returns included in the ARIMA-GARCH models illustrates that the returns obtained are insufficient to compensate for the additional risk assumed by the variable income pension funds. Damián (2016) works on the beginnings of the new pension system in Mexico, demonstrating the reduction in the benefits of future annuitants due to this reform, and analyzes the poverty it causes in the population at retirement age, concerning contributory and non-contributory

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pensions. Bernal (2016) examines the factors that explain pension spending and projects its evolution to the year 2075 in four Latin American countries, Chile, Peru, Colombia, and Mexico. Bernal finds that pension spending for GDP is between 1.8 and 6.4% but will double once and then four more times by 2075, mainly due to the aging of the population. Lastly, Castañón and Ferreira (2017) show results of contribution density considering different cohorts of workers contributing to the Retirement Fund Administrators (Spanish: Administradora de Fondos para el Retiro, Afore). They conclude that the contributions of such workers are low and are insufficient for a decent pension.

For governments, the estimation of the amount of the actuarial reserve necessary to cover all the payments of an annuity directly affects the coverage and stability of the pension systems and, therefore, the fiscal sustainability of the countries that grant or subsidize the annuities of the population. An imbalance in this system affects the sustainability of the entities in charge of pension obligations, with probable systemic destabilizations in the economy (Grinols & Turnovsky,1993; Schmedders, 1998). In the case of Mexico, Banda and Gómez (2009) analyze the performance of Mexican Specialized Investment Companies for Retirement Funds (Spanish: Sociedades de Inversión Especializadas en Fondos para el Retiro en México, Siefores) using the Sharpe, Treynor, and Jensen indices. Banda and Gómez conclude that there is little relationship between market returns and the returns of Afores and that the risk associated with Siefores 3, 4, and 5 is expected to increase as well as the returns of workers, which are very low.

The actuarial calculation proposes that the estimate of the cash reserves necessary to support the payment of an annuity is made by adding the expected present values of future payments (Bowers et al., 1997). This calculation involves several variables, such as life expectancy, inflation, age, the amount of the possible future payment, and the discount interest rate. This estimate does not include the stochastic structure of the assets included in the portfolio or the total portfolio in which the actuarial reserve is invested. Therefore, the analyses are framed in a deterministic world.

Background and objectives

The literature widely discusses portfolio management methodologies based on hedging through financial derivatives. Especially since the publication of the Black and Scholes (1973) and Merton (1973) models, these strategies are commonly used in financial markets both for equities, as indicated by Jarrow and Turnbull (1999), and for bonds and other fixed-income securities (Jarrow, 2002). Strategies of this type have also been applied in the context of commodity markets; for example, in the case of electric power (Nässäkkälä & Keppo, 2005). In several articles, the study of financial phenomena has been approached through the stochastic dynamics of prices. Some of the works with this type of approach are Merton

(1973), Black and Scholes (1973), Vasicek (1977), Cox and Ross (1976), Cox et al. (1985a and 1985b), Hull and White (1990 and 1993), Black et al. (1990), and Heath et al. (1992).

This work aims to develop a model to estimate the actuarial reserve of a life annuity using stochastic processes. It is based on a dynamic hedging strategy associated with a financial derivative. In this dynamic, said hedging guarantees a future value of the reserve for each disbursement greater than, or at least equal to, the payment to be made. Section 3 presents the model, and Section 4 analyzes the results of its implementation in the Pension Fund of the Territorial Entities of Colombia and the retirement funds of workers affiliated with the Mexican Social Security Institute (Spanish: Instituto Mexicano del Seguro Social, IMSS) and the Institute of Security and Social Services for State Workers (Spanish: Instituto de Seguridad y Servicios Sociales de los Trabajadores del Estado, ISSSTE) in Mexico. Finally, Section 5 offers the conclusions.

Methodology

This section proposes the model based on the estimation of the actuarial reserve necessary to support the payment of an annuity and the breakdown of the payer's obligation into a long call option and a short put option for each of the future disbursements. Subsequently, an equation is presented to estimate the present value of the annuity surplus of the payer when the value set aside exceeds the amount payable. Moreover, this study presents a dynamic hedging strategy, which, through a synthetic put option, makes it possible to cancel any deficit in the actuarial reserve for each payment, generating surpluses in most cases. These surpluses are valued using the Cox, Ingersoll, and Ross (1985b) model and are subtracted from the initial calculation of the required reserve, together with the costs generated by the dynamic hedging (or added if they are revenues).

Reserve required for a life annuity calculated on a life annuity basis

An annuity is a periodic payment of a pension during the annuitant's life; its calculation depends on the annuitant's life expectancy. The following sections determine the elements necessary to calculate the amount of money needed to support the payment of a life annuity using the proposed dynamic hedging.

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Survival probability

$S(x)$ is a survival function that indicates the likelihood that the age at the time of death (X) of a specific person is greater than age x. Therefore:

$$S(x) = P(X > x) \tag{1}$$

${}_tp_x$ is defined as the probability that a person of x age has of not dying in t years and ${}_tq_x$ is the probability of dying in the same period. The following formulas derive from the definition above:

$${}_tp_x = P(X > x + t | X > x) \tag{2}$$

$${}_tp_x = \frac{P(X > x \cap X > x + t)}{P(X > x)} \tag{3}$$

$${}_tp_x = \frac{P(X > x + t)}{P(X > x)} \tag{4}$$

$${}_tp_x = \frac{S(x + t)}{S(x)} \tag{5}$$

These probabilities can be calculated through mortality tables, which are models of survival presented in tabular form and illustrate the number of survivors of an initial group, denoted as l_0, from a particular age x up to the age limit w. l_0 is the root of the initial table or group and l_x is the estimated number of survivors for age x. The following equation determines the relationship between the constant l_0 and variable l_x:

$$l_x = S(x) * l_0 \tag{6}$$

as, ${}_tp_x = \frac{S(x+t)}{S(x)}$ y $l_x = S(x) * l_0$, then,

$$S(x) = \frac{l_x}{l_0} \tag{7}$$

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therefore,

$$ {}_{t}p_{x} = \frac{l_{x+t}}{l_{x}} \tag{8} $$

Actuarial reserve calculation

The reserve necessary for the payment of an annuity is the sum of the expected present values of the possible payments (Bowers et al., 1997). It assumes an annuity with an indemnity that increases by the same percentage as the general price level and that initially corresponds to a value of D_0 if the person lives and (0) if they die.

$$ R_N = E(\text{Payment}_0)(1+r)^{-0} + E(\text{Payment}_1)(1+r)^{-1} + E(\text{Payment}_2)(1+r)^{-2} + \ldots + E\left(\text{Payment}_{(w-\phi)}\right)(1+r)^{-(w-\phi)} \tag{9} $$

$$ R_N = \left(D_0\ {}_0p_x + 0\ {}_0q_x\right)(1+r)^{-0} + \left(D_1\ {}_1p_x + 0\ {}_0q_x\right)(1+r)^{-1} + \left(D_2\ {}_2p_x + 0\ {}_0q_x\right)(1+r)^{-2} + \cdots + \left(D_{(w-x)}\ {}_{(w-x)}p_x + 0\ {}_{(w-x)}q_x\right)(1+r)^{-(w-x)} \tag{10} $$

where:

D_t: the amount of allowance in a moment t

r: nominal interest rate

x: current age of the person insured

$w - x$: maximum time that the insured person can live

R_N: reserve required for annuity payments.

Therefore,

$$ R_{Nt} = \sum_{t=0}^{w-x} D_t\ {}_tp_x(1+r\%)^{-t} \tag{11} $$

In continuous time it can be expressed as follows:

$$R_{Nt} = \int_0^{w-x} D_t \, {}_tp_x e^{-rt} dt \tag{12}$$

Combination of options to represent the annuity payment

V_t is the value of the fund required to support the future payment D_T, which is executed if and only if the person of x age survives for at least $T - t$ more years. Therefore, the value of such fund corresponds to the actuarial present value:

$$V_t = D_T \, {}_{(T-t)}p_x e^{-r(T-t)} \tag{13}$$

where r is the nominal technical interest rate used for the actuarial calculation and ${}_{(T-t)}p_x$ is the probability that a person of x age survives for at least $T - t$ more years. The assumption is that V_t is driven by the following Stochastic Differential Equation (SDE):

$$dV_t = rV_t dt + \sigma_v V_t dW_t \tag{14}$$

If $V_T < D_T$, this would default on the payment or take money from the reserve for another payment, thus defunding the total reserve. Moreover, there would be no surplus of any kind, which is denoted as S_T, in the portfolio backing the life annuity. The above means that $S_T = 0$. On the other hand, if $V_T \geq D_T$ payment is then fulfilled, and therefore the surplus would be $S_T = V_T - D_T$. Consequently:

$$S_T = max(V_T - D_T, 0) \tag{15}$$

This corresponds to payment on the due date T of a call. Based on the equations of Black and Scholes (1973) and Merton (1973), and assuming that the annuity reserve has a return, denoted by r, equal to the risk-free rate:

$$S_t = V_t \Phi(d_{1,t}) - D_T e^{-r(T-t)} \Phi(d_{2,t}) \tag{16}$$

where:

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$$d_1 = \frac{Ln\left(\frac{V_t}{D_T}\right) + \left(r + \frac{1}{2}\sigma_v^2\right)(T-t)}{\sigma_v\sqrt{T-t}}$$

(17)

$$d_2 = d_1 - \sigma_v\sqrt{T-t}$$

(18)

and $\Phi(x)$ is the probability that a normally distributed variable with a zero mean and standard deviation of one will be smaller than x.

From $V_t = D_{T\,(T-t)}p_x e^{-r(T-t)}$, the following is derived:

$$d_1 = \frac{Ln\left({}_{(T-t)}p_x e^{-r(T-t)}\right) + \left(r + \frac{1}{2}\sigma_v^2\right)(T-t)}{\sigma_v\sqrt{T-t}}$$

(19)

If $V_T < D_T$, the shortfall is given by $F_T = D_T - V_T$. Otherwise, the shortfall is zero ($F_T = 0$). Therefore:

$$F_T = max(D_T - V_T, 0)$$

(20)

and

$$F_t = D_T e^{-r(T-t)}\Phi\left(-d_{2,t}\right) - V_t\Phi\left(-d_{1,t}\right)$$

(21)

with

$$d_1 = \frac{Ln\left({}_{(T-t)}p_x e^{-r(T-t)}\right) + \left(r + \frac{1}{2}\sigma_v^2\right)(T-t)}{\sigma_v\sqrt{T-t}}$$

(22)

$$d_2 = d_1 - \sigma_v\sqrt{T-t}$$

(23)

Since the payer of the obligation must cover the possible shortfall with their resources or keep the potential surplus, then there is a call option on the long position and a put option on the short position, both with no underlying cost V_t and strike price D_T. Neftci (2008) developed a similar strike to the one described above, creating a long-put option on a long position from a long call and a short sale of the underlying asset.

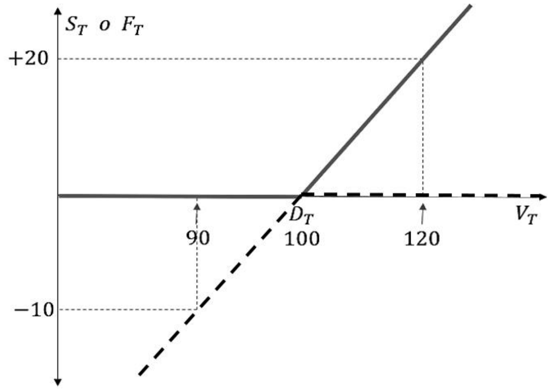

Figure 1. Surplus or deficit of an annuity for a pension payer and its breakdown as a long call option and a short put option.
Source: created by the author, R-Project.

Therefore, the time value of the option portfolio t satisfies the put-call parity condition:

$$S_t - F_t = V_t - D_T * e^{-r*(T-t)} \tag{24}$$

The gain obtained is the difference between V_t and the payment D_T to be made, discounted at a rate of return equal to the trend parameter of the SDE describing the change in V_t.

Valuation of S_t based on a credit risk model

Following Merton (1974), cited by Venegas (2008), it is possible to calculate the probability of total or partial default on payment D_T, based on the following deduction, if $S_T = \max(V_T - D_T, 0)$, then:

$$S_t = e^{-r(T-t)} \int_{-\infty}^{\infty} S_T f_{V_T|V_t}(v|V_t) dv \tag{25}$$

$$S_t = e^{-r(T-t)} \int_{-\infty}^{\infty} \max(v - D_T, 0) f_{V_T|V_t}(v|V_t) dv \tag{26}$$

$$S_t = e^{-r(T-t)} \int_D^{\infty} v f_{V_T|V_t}(v|V_t) dv - D_T e^{-r(T-t)} \int_{\{v>D_T\}} f_{V_T|V_t}(v|V_t) dv \tag{27}$$

$$S_t = e^{-r(T-t)} E\left[V_{T\{v>D_T\}} \middle| V_t\right] - D_T e^{-r(T-t)} \mathbb{P}\{V_T > D_T | V_t\} \tag{28}$$

where $f_{V_T|V_t}(v|V_t)$ is the probability density function of V_T, conditional on the initial v_t value.

In this case, the probability that there will be no default in the future payment D_T, given the present value of the reserve for such payment V_t is $\mathbb{P}\{V_T > D_T|V_t\}$.

Reserve deficits and dynamic coverage

In order to determine the amount of the reserve created, the obligation must be considered as the combination of a long call option with a short put option. Thus, a dynamic hedge involves finding an optimal portfolio in which market risk is eliminated when $V_T < D_T$. The portfolio is considered Π_t consisting of w_1 units of short put options in the reserve V_t and w_2 put options (F_t) in a short position.

$$\Pi_t = -w_1 V_t - w_2 F_t \tag{29}$$

The change in the value of the portfolio is given by

$$d\Pi_t = -w_1 dV_t - w_2 dF_t \tag{30}$$

Itô's Lemma provides the DTS describing the change in F_t.

$$dF_t = \left(\frac{\partial F}{\partial t} + \frac{\partial F}{\partial V_t} rV_t + \frac{1}{2}\sigma_v^2 V_t^2 \frac{\partial^2 F}{\partial V_t^2}\right) dt + \frac{\partial F}{\partial V_t}\sigma_v V_t dW_t \tag{31}$$

replacing dV_t and dF_t in $d\Pi_t$ the following is obtained:

$$d\Pi_t = \left(-w_1 - w_2 \frac{\partial F}{\partial t}\right) rV_t dt + \left(-w_1 - w_2 * \frac{\partial F}{\partial V_t}\right)\sigma_v V_t dW_t - w_2\left(\frac{\partial F}{\partial t} + \frac{1}{2}\sigma_v^2 V_t^2 \frac{\partial^2 F}{\partial V_t^2}\right) dt \tag{32}$$

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In order to eliminate market risk, if $w_2 = 1$ is done, it is necessary that $-w_1 = \frac{\partial F}{\partial V_t}$, and because $F_t = D_T e^{-r(T-t)} \Phi(-d_{2,t}) - V_t \Phi(-d_{1,t})$, then:

$$\frac{\partial F}{\partial V_t} = -1 + \Phi(d_{1,t}) \tag{33}$$

therefore,

$$w_1 = 1 - \Phi\left(\frac{\mathrm{Ln}\left(\frac{V_t}{D_T}\right) + \left(r + \frac{1}{2}\sigma_v^2\right)(T-t)}{\sigma_v\sqrt{T-t}}\right) \tag{34}$$

That is, for each short put option contained in the portfolio (or equivalently, for each obligation to pay D_T in time T), in moment t a percentage of the reserve should be held in short position V_t equal to $1 - \Phi(d_{1,t})$. If such a position is not possible, an asset Z is used so that $\mathrm{Corr}(V_t, Z_t) \approx 1$. The above strategy constitutes a delta hedge for a short put option.

The risk assumed in a succession of payments D_T, with $T = 1,2,3, \ldots, n$, is equivalent to having a portfolio consisting of call options in a long position at no cost, whose individual valuation is given simultaneously by the following two equations, already discussed in sections 2.2 and 2.3 respectively (equations 16 and 28)

$$S_t = V_t \Phi(d_{1,t}) - D_T e^{-r(T-t)} \Phi(d_{2,t}) \tag{35}$$

and

$$S_t = e^{-r(T-t)} \mathrm{E}\left[V_{T\{v>D_T\}} \middle| V_t\right] - D_T e^{-r(T-t)} \mathbb{P}\{V_T > D_T | V_t\} \tag{36}$$

From the above, it can be deduced that

$$V_t \Phi(d_{1,t}) = e^{-r(T-t)} \mathrm{E}\left[V_{T\{v>D_T\}} \middle| V_t\right] \tag{37}$$

the conditional expected value of the reserve for due date T, is given by:

$$\mathrm{E}\left[\mathrm{V}_{\mathrm{T}\{\mathrm{v}>\mathrm{D}_{\mathrm{T}}\}}\middle|\mathrm{V}_{\mathrm{t}}\right] = \mathrm{V}_{\mathrm{t}}\Phi\left(\mathrm{d}_{1,\mathrm{t}}\right)\mathrm{e}^{\mathrm{r}(\mathrm{T}-\mathrm{t})} \tag{38}$$

Bond pricing (Cox-Ingersoll-Ross)

Assuming that the interest rate of the economy follows a process:

$$\mathrm{dr} = \mathrm{a}(\mathrm{b}-\mathrm{r})\mathrm{dt} + \sigma_{\mathrm{r}}\sqrt{\mathrm{r}}\mathrm{dW}_{\mathrm{t}} \tag{39}$$

Then, the price of a zero-coupon bond is determined by the following set of equations proposed by Cox, Ingersoll, and Ross (1985b).

$$\mathrm{B}(\mathrm{r},\mathrm{t};\mathrm{T}) = \mathrm{e}^{\mathrm{A}(\mathrm{t};\mathrm{T})-\mathrm{rD}(\mathrm{t};\mathrm{T})} \tag{40}$$

$$\mathrm{A}(\mathrm{t};\mathrm{T}) = \ln\left[\left(\frac{2\sqrt{\mathrm{a}^2+2\sigma_{\mathrm{r}}^2}\,\mathrm{e}^{\left(\mathrm{a}+\sqrt{\mathrm{a}^2+2\sigma_{\mathrm{r}}^2}\right)(\mathrm{T}-\mathrm{t})/2}}{\left(\mathrm{a}+\sqrt{\mathrm{a}^2+2\sigma_{\mathrm{r}}^2}\right)\left(\mathrm{e}^{\sqrt{\mathrm{a}^2+2\sigma_{\mathrm{r}}^2}(\mathrm{T}-\mathrm{t})}-1\right)+2\sqrt{\mathrm{a}^2+2\sigma_{\mathrm{r}}^2}}\right)^{2\mathrm{ab}/\sigma_{\mathrm{r}}^2}\right] \tag{41}$$

$$\mathrm{D}(t;T) = \frac{2\left(e^{\sqrt{a^2+2\sigma_r^2}(T-t)}-1\right)}{\left(a+\sqrt{a^2+2\sigma_r^2}\right)\left(e^{\sqrt{a^2+2\sigma_r^2}(T-t)}-1\right)+2\sqrt{a^2+2\sigma_r^2}} \tag{42}$$

Since the proposed dynamic hedging generates surpluses in the reserve (savings generated), the proposal is that these cash flows be valued using the model above. This valuation makes it possible to quantify these future flows at present value, thus making it possible to adjust the actuarial reserve necessary to support the life annuity payment.

Savings generated

Through dynamic hedging, it is theoretically impossible for the actuarial reserve for the payment to be less than the payment, i.e., $V_T < D_T$. Therefore, in the worst case V_T is equal to D_T and in the best case, a surplus of money arises. The present value of these surpluses is the savings in the total actuarial reserve resulting from the hedge. This saving, denoted Ah, is equal to the value of a bond whose coupons correspond to the surplus in each period. This value can be estimated using the Cox, Ingersoll, and Ross (1985b) short rate model from Section 2.4.

$$Ah = \int_0^{w-x} E\left(V_{T\{v>D_T\}} \middle| V_t\right) B(r,t,T) dT \tag{43}$$

$$Ah = \int_0^{w-x} V_t \Phi\left(d_{1,t}\right) e^{r(T-t)} e^{A(t;T)-rD(t;T)} dT \tag{44}$$

Revenues/costs associated with dynamic hedging

The proposed dynamic hedging, carried out for each payment, generates revenue or a cost derived from the same operation. In the zero time ($t = 0$), this income or cost is equal to the value of the portfolio at that moment multiplied by the number of units sold, which in this case is delta at zero (0), minus the cost associated with the interest rate of the credit. This is:

$$I_0 = V_0 U_0 - C_0 \tag{45}$$

Like $U_0 = \Delta_0$ y $C_0 = V_0 \Delta_0 \frac{i\%}{252}$, then:

$$I_0 = V_0 \Delta_0 \left(1 - \frac{i\%}{252}\right) \tag{46}$$

On the other hand, the income/cost at the moment of t is:

$$I_t = I_{t-1}\left(1 + \frac{i\%}{252}\right) + V_t U_t - C_t \tag{47}$$

$$I_t = I_{t-1}\left(1 + \frac{i\%}{252}\right) + V_t U_t - I_{t-1} V_t U_t \frac{i\%}{252} \tag{48}$$

like $U_t = \Delta_{t-1} - \Delta_t$, then

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$$I_t = I_{t-1}\left(1+\frac{i\%}{252}\right) + V_t(\Delta_{t-1}-\Delta_t) - I_{t-1}V_t(\Delta_{t-1}-\Delta_t)\frac{i\%}{252} \tag{49}$$

Therefore, the income/cost over time $T-1$ is:

$$I_{T-1} = I_{T-2}\left(1+\frac{i\%}{252}\right) + V_{T-1}(\Delta_{T-2}-\Delta_{T-1}) - I_{T-2}V_{T-1}(\Delta_{T-2}-\Delta_{T-1})\frac{i\%}{252} \tag{50}$$

And in time T:

$$I_T = V_T\int_0^{T-1}(\Delta_{k-1}-\Delta_k)dk \tag{51}$$

then, the total revenue or cost is:

$$(I/C)_T = I_{T-1} - I_T \tag{52}$$

If the result is negative, it is understood that the hedging generates cost, not income.

Generated Profit (UT)

The dynamic hedging realized for each possible payment has an income or cost priced using the short rate model in section 2.5 (Bond pricing).

$$UT = \int_0^{w-x}(I/C)_T B(r,t,T)dT \tag{53}$$

$$UT = \int_0^{w-x}(I/C)_T e^{A(t;T)-rD(t;T)}dT \tag{54}$$

Necessary reserve

The amount of money needed to support the payment of an annuity using the dynamic hedging is:

$$R_N = \int_0^{w-x} D_{T(T-t)} p_x e^{-r^*(T-t)} dT - Ah - UT \tag{55}$$

Results of the case studies

The pension system in force in Colombia since 1993 (Law 100) was created as a measure to put an end to the multiple pension systems that existed up to that time. Today, for the majority of the Colombian population, two regimes coexist, the Individual Savings with Solidarity Regime (Spanish: Régimen de Ahorro Individual con Solidaridad, RAIS), which is the scheme of the private pension fund administrators (Spanish: Administradoras de Fondos de Pensiones privadas, AFP), and the Average Premium with Defined Benefit Solidarity Regime (Spanish: Régimen de Prima Media, RPM), which is operated by the public administrator Colpensiones. However, annuities are still paid to people who contributed to other regimes, such as those offered by territorial entities (municipalities and departments), whose actuarial reserves do not yet equal the pension liabilities of these administrative units. Likewise, in the Mexican case, the change to the new pension system of Law 97 leaves the government with a deficit in the pensions of workers affiliated with the previous law since it is a defined benefit system. Therefore, the burden of paying such pensions will be borne by the federal government.

This article is relevant due to the above, and it determines a solution to the deficit that these nations could have for the payment of their pensions.

Pension fund of the territorial entities of Colombia

In Colombia, the Pension Fund for Territorial Entities (Spanish: Fondo de Pensiones de las Entidades Territoriales, FONPET) is a fund established by the Ministry of Finance and Public Credit to support the payment of pensions to former employees of the territorial divisions of the country. FONPET is defined as "a fund without legal status administered by the Ministry of Finance and Public Credit, aiming to collect and allocate resources to the accounts of territorial entities and manage resources through autonomous assets" (Directorate of Investments and Public Finance, 2017). FONPET currently manages approximately USD 16 billion, making it the largest and most significant fund in the country. The financial stability of the departments, municipalities, and other governmental entities depends to a large extent on the proper administration of this fund.

Figure 2 shows the evolution of the FONPET unit price from February 1, 2018, to January 31, 2019. During this period, its annual profitability was $\hat{\mu}_v = 3.81\%$ and its annual volatility $\hat{\sigma}_v = 1.19\%$. The stochastic differential equation that follows the unit price is:

$$dV_t = 0{,}0381V_t dt + 0{,}0119V_t dW_t$$

(56)

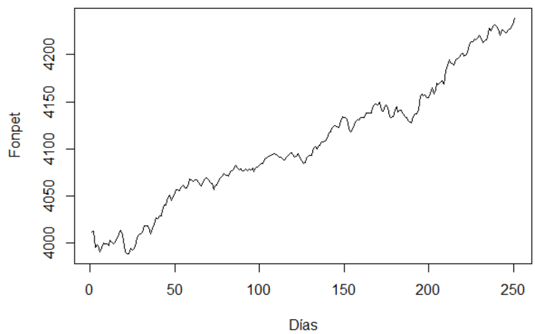

Figure 2. FONPET unit value (01/02/2018-31/01/2019).
Source: created by the author based on data from FONPET, R-Project.

This section uses the official mortality tables for Colombia shown in Annex 1 and considers a 60-year-old man whose monthly payments (pension allowances) grow annually by 3.5%, with the first annual payment corresponding to USD 7,000; likewise, a woman with the same characteristics was considered. An actuarial calculation using the methodology proposed by Bowers et al. (1997) yields an actuarial reserve value of USD 156,467 with a technical interest rate of 3.81% nominal annual interest for men and USD 182,484 for women.

Assuming that the interest rate of the 10-year bond, issued by the Ministry of Finance and Public Credit of Colombia, follows a Cox-Ingersoll-Ross (CIR) process, an estimation of the parameters is obtained. For this purpose, the Hessian method, exposed by Remillard (2016), and the est.cir function of R-project are used.

$$dr = a(b - r)dt + \sigma_r \sqrt{r} dW_t$$

(57)

$$dr = 0{,}04919(0{,}063892 - r)dt + 0{,}006308\sqrt{r} dW_t$$

(58)

After 1,000 iterations, it is possible to observe that in all cases, there is a decrease in the capital required to finance the life annuity as well as the actuarial reserve required for each iteration performed by gender. Figure 3 illustrates the above.

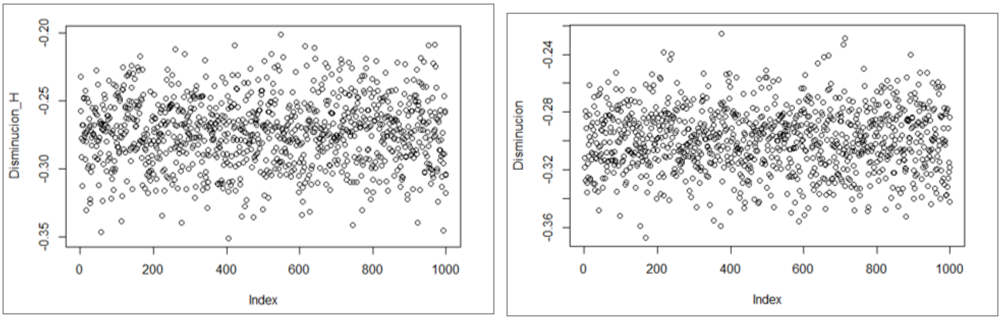

Figure 3. Decrease, by gender, in the actuarial reserve required for each iteration (men on the left, women on the right).
Source: created by the author, R-Project.

The lowest decrease is 20.09% for men, and the highest decrease is 35.11%; for women, the lowest decrease is 22.54%, and the highest decrease is 36.73%, as presented in Table 1.

Table 1
Minimum, first quartile, median, mean, third quartile, and maximum savings generated by the proposed model

Gender	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Men	-0.3511185	-0.2893935	-0.2726112	-0.273163	-0.2570137	-0.2009618
Women	-0.3672644	-0.3135634	-0.2992640	-0.29937	-0.2846892	-0.2253949

Source: created by the author, R-Project

The average decrease is 27.32% for men, representing a saving of USD 42,741, going from an actuarial reserve of USD 156,467 to USD 113,726. The average decrease is 29.93% for women, going from an actuarial reserve of USD 182,484 to USD 127,854, representing a saving of USD 54,630.

Figure 4 presents the boxplot for the same variable.

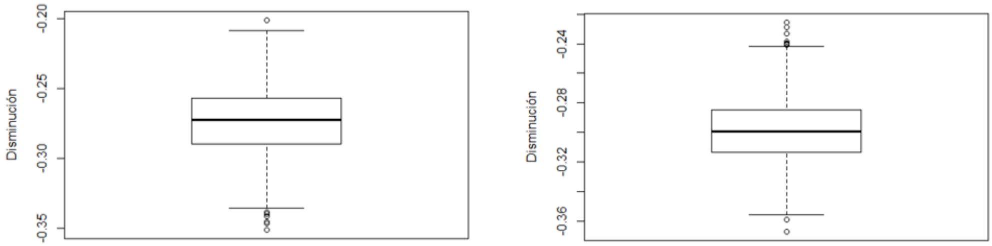

Figure 4: Box plot of the decrease, by gender, in the actuarial reserve.
Source: created by the author, R-Project.

It can be observed that less than 20.84% decreases in profitability for men and 24.14% for women are considered higher outliers. Those above 33.8% for men and 35.69% for women are considered lower outliers.

Figure 5 shows the histogram of frequencies of decreases in required reserves for both men and women.

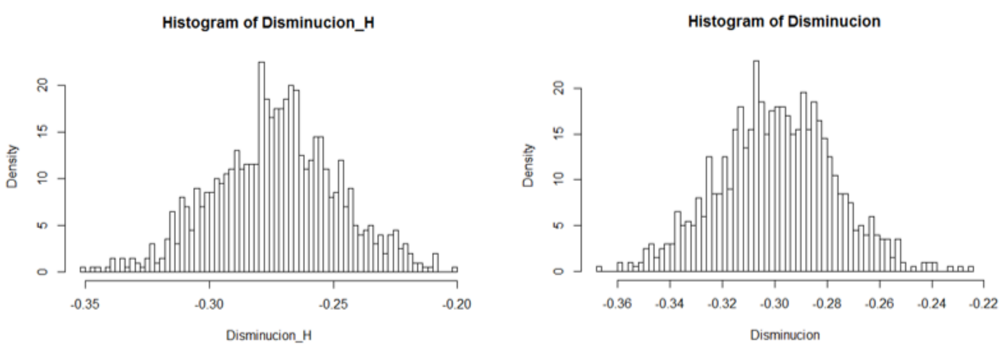

Figure 5. Frequency histogram of decreases per iteration.
Source: created by the author, R-Project.

As of December 2018, the actuarial reserve of this fund was approximately USD 16 billion, and the pension liability borne by territorial entities exceeded USD 22 billion. Assuming that the fund managers implement the strategy presented in this research, there is a decrease of 27.32% for men and 29.94% for women (average decrease according to the 1,000 scenarios). Besides, the proportion of men and women in the participation of the fund was equal to that of the general population (49.37% men and 50.63% women), so the pension liability would be USD 11,417 million, which would free up resources to cover other needs of the population for USD 4,583 million.

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Retirement funds for workers affiliated with IMSS and ISSSTE in Mexico

The 1973 Mexican Social Security Institute (IMSS) Law presented a defined benefit pension scheme, characterized by a government that assumes the deficit of the accounts that fail to accumulate the actuarial reserve necessary to pay a life annuity. After more than twenty years, the Mexican government modified the pension system to increase the country's domestic savings using social security funds. Until 1992, the pension system for Mexicans affiliated with the IMSS and the Institute of Security and Social Services for State Workers (ISSSTE) was based on a defined benefit scheme; that is, they knew in advance how much they were going to receive for their pensions when they retired.

Due to demographic changes indicating increasing life expectancy and declining birth rates, coupled with high pension promises and low worker contribution rates, such pension plans could not be sustained. Thus, on December 21, 1995, the new Social Security Law was published in the Official Gazette of the Federation (Spanish: Diario Oficial de la Federación, DOF), Congress (1995), with a defined contribution scheme, in which it is established that each worker will have a fund in an individual account in a Retirement Fund Administrator (Spanish: Administradora de Fondos para el Retiro, Afore), through its Specialized Investment Companies for Retirement Funds (Siefore), and that each worker must accumulate in their account during their working life (up to the age of 65) the actuarial reserve sufficient to sign up for a life annuity with an insurance company. In other words, workers enjoy a benefit equivalent to what they manage to save. With this law, there is a possibility that workers who have contributed 1,250 weeks may not accumulate enough to sign up for such a pension plan. In this case, the Mexican government will subsidize a guaranteed minimum pension (one minimum wage). Workers who want to enjoy a life pension and do not have the necessary resources at the time of retirement will have to carry the deficit in their account to purchase their life pension.

Figure 6 presents the evolution of the index value of Siafore Básica 1, the portfolio in which the Afores invest the actuarial reserves of people over 60 years of age. This information corresponds to the period from February 1, 2018, to January 31, 2019. During this period, annual profitability was $\hat{\mu}_v = 4.4151\%$, and the annual volatility was $\hat{\sigma}_v = 2.4981\%$, so that the stochastic differential equation that follows the unit price is:

$$dV_t = 0.044151 V_t dt + 0.024981 V_t dW_t \tag{59}$$

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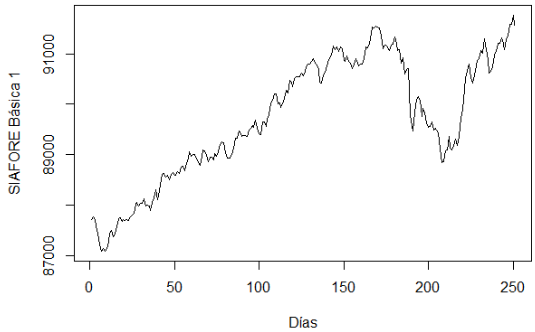

Figure 6. Index value of Siefore Básica 1 (01/02/2018-31/01/2019).
Source: created by the author with data from CONSAR, R-Project.

This section uses the official mortality tables for Mexico presented in Annex 2 and considers a 60-year-old man whose monthly payments (pension allowances) grow annually by 3.5%. The first annual payment corresponds to USD 7,000. Similarly, the study considers a woman with the same characteristics. The traditional actuarial calculation yields an actuarial reserve value of USD 152,296 for men and USD 174,348 for women with a technical interest rate of 4.4151% nominal annual rate. Assuming that the interest rate of the 10-year bond issued by Banco de México follows a Cox-Ingersoll-Ross (CIR) process, the study obtains an estimate of the parameters through the Hessian method, Remillard (2013), and the R-project est.cir function.

$$\mathrm{dr} = \mathrm{a}(\mathrm{b} - \mathrm{r})\mathrm{dt} + \sigma_{\mathrm{r}}\sqrt{\mathrm{r}}\mathrm{dW_t} \tag{60}$$

$$\mathrm{dr} = 0{,}012831(0{,}061129 - \mathrm{r})\mathrm{dt} + 0{,}005712\sqrt{\mathrm{r}}\mathrm{dW_t} \tag{61}$$

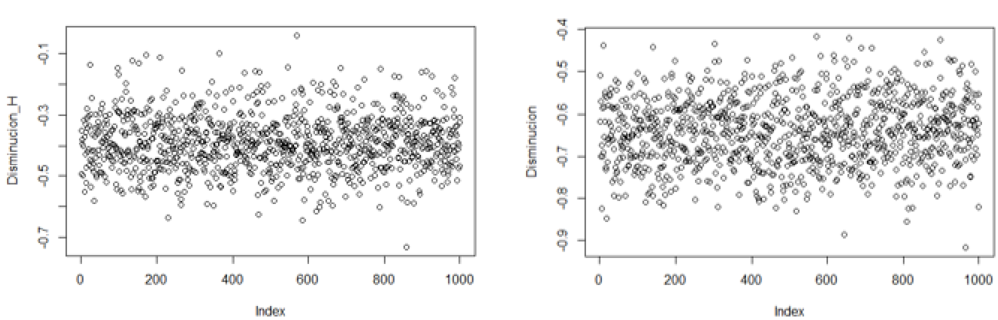

Figure 7. Decrease in actuarial reserve required for each iteration by gender (men on the left, women on the right).
Source: created by the author, R-Project.

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After 1,000 iterations, it is evident that there is a decrease in the capital required to finance the life annuity (Figure 7). These iterations also reveal the decrease, by gender, in the actuarial reserve required for each iteration performed.

For men, the smallest decrease is 4%, and the largest is 73.27%; for women, the smallest decrease is 41.65%, and the largest is 91.66%, as illustrated in Table 2.

Table 2
Minimum, first quartile, median, mean, third quartile, and maximum savings generated by the proposed model

Gender	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Men	-0.7327453	-0.4505040	-0.3946375	-0.390327	-0.3350959	-0.0399676
Women	-0.9165761	-0.6987917	-0.6411653	-0.641158	-0.5846981	-0.4165424

Source: created by the author, R-Project

The average decrease for men is 39.03%, representing a saving of USD 59,445, going from an actuarial reserve of USD 152,296 to one of USD 92,851. The average decrease is 64.12% for women, from a reserve of USD 174,348 to USD 62,564, representing a decrease of USD 111,785. Figure 8 presents the boxplot for the same variable.

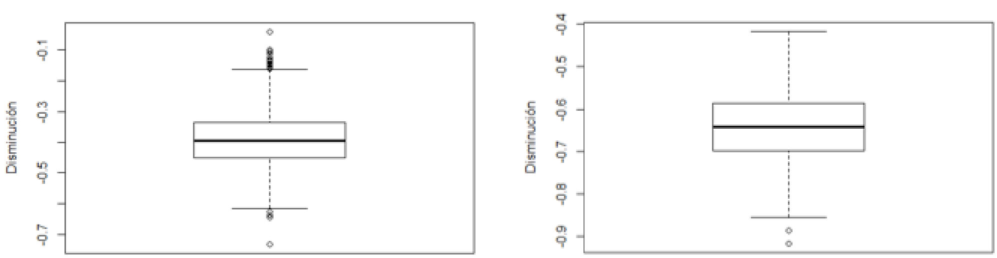

Figure 8. Box plot of the decrease in actuarial reserve by iteration by gender (men on the left).
Source: created by the author, R-Project.

As can be seen, decreases lower than 16.20% and higher than 62.36% for men are considered outliers. For women, those above 87% are considered outliers. Figure 9 presents the frequency histogram.

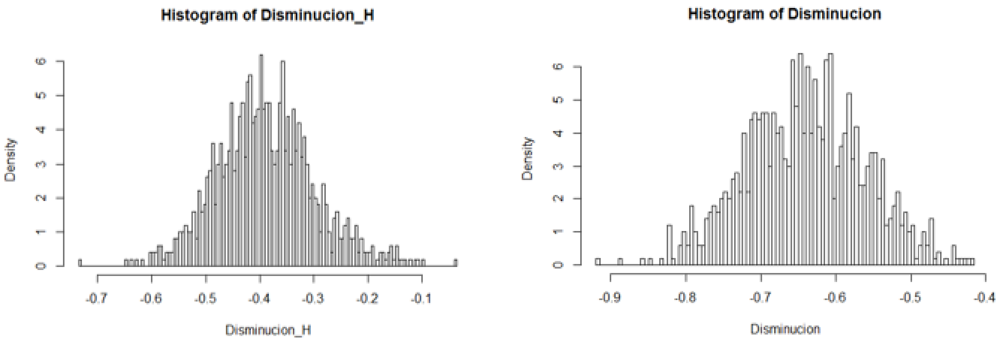

Figure 9. Histogram of frequencies of decreases per iteration by gender (men on the left, women on the right).
Source: created by the author, R-Project.

According to the National Commission of the Retirement Savings System (Spanish: Comisión Nacional del Sistema de Ahorro para el Retiro, CONSAR) (2018), public expenditure allocated to the Minimum Guaranteed Pension (Spanish: Pensión Mínima Garantizada, PMG) comes to, in present value, USD 43,697 million. Assuming that the retirement fund managers implement the strategy, there is a decrease in the actuarial reserve requirement of 39.03% for men and 64.12% for women. The gender composition of pension rights holders is equal to the gender composition of the general population (48.6% men and 51.40% women); the pension liability would be USD 21,007 million, representing a saving of USD 22,690 million.

Conclusions

This research proposes a model for estimating the amount of the actuarial reserve of a life annuity. This approach considers the stochastic dynamics of the assets in which this reserve is invested and, therefore, the portfolio supporting the annuity. A dynamic hedging strategy is also presented, which, through a synthetic put option, cancels any deficit in the actuarial reserve, making it theoretically impossible for the annuity payer to suffer losses. The surpluses generated are valued using the Cox, Ingersoll, and Ross (1985b) model and are subtracted from the initial calculation of the required reserve, together with the costs generated by the dynamic hedge (or added if they are revenues).

The proposed model is applied to FONPET in Colombia and the retirement funds of workers affiliated to IMSS and ISSSTE in Mexico. In the first case, the proposed portfolio management strategy could represent savings in public spending of 28.64%, equivalent to approximately USD 4,583 million. In the case of Mexico, savings are 51.92%, equivalent to approximately 22.69 billion US dollars. The relevance of the strategy lies in the possibility of reducing the cost of an annuity, allowing greater hedging of the retirement systems, with the social and tax benefits that this implies.

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Annex

Table A1
Mortality table for Colombia

x	l(x) men	l(x) women	x	l(x) men	l(x) women	x	l(x) men	l(x) women
15	1.000.000	1.000.000	47	966.561	981.111	79	599.994	753.202
16	999.515	999.728	48	964.017	979.663	80	570.538	731.048
17	999.019	999.450	49	961.269	978.097	81	539.892	707.105
18	998.510	999.165	50	958.298	976.402	82	508.181	681.314
19	997.988	998.872	51	955.085	974.566	83	475.562	653.637
20	997.451	998.570	52	951.608	972.576	84	442.222	624.065
21	996.898	998.259	53	947.843	970.418	85	408.381	592.620
22	996.327	997.938	54	943.766	968.077	86	374.288	559.368
23	995.736	997.606	55	939.348	965.536	87	340.219	524.423
24	995.124	997.262	56	934.604	962.801	88	306.474	487.954
25	994.488	996.905	57	929.498	959.851	89	273.371	450.192
26	993.826	996.533	58	923.991	956.662	90	241.235	411.435
27	993.136	996.145	59	918.039	953.206	91	210.391	372.049
28	992.415	995.740	60	911.595	949.454	92	181.152	332.340
29	991.660	995.315	61	904.607	945.372	93	153.808	292.640
30	990.868	994.869	62	897.019	940.925	94	128.609	253.452
31	990.036	994.400	63	888.769	936.072	95	105.758	215.411

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32	989.159	993.906	64	879.635	930.769	96	85.395	179.222
33	988.233	993.384	65	869.557	924.968	97	67.556	145.594
34	987.254	992.832	66	858.477	918.617	98	52.206	115.166
35	986.216	992.247	67	846.334	911.658	99	39.285	88.438
36	985.114	991.625	68	833.069	904.029	100	28.688	65.719
37	983.942	990.963	69	818.623	895.662	101	20.255	47.092
38	982.693	990.258	70	802.940	886.485	102	13.771	32.413
39	981.360	989.505	71	785.968	876.420	103	8.975	21.338
40	979.936	988.699	72	767.658	865.384	104	5.580	13.370
41	978.411	987.836	73	747.970	853.289	105	3.290	7.930
42	976.776	986.910	74	726.872	840.044	106	1.828	4.425
43	975.021	985.916	75	704.342	825.554	107	950	2.306
44	973.135	984.846	76	680.372	809.722	108	458	1.112
45	971.105	983.694	77	654.970	792.450	109	202	492
46	968.919	982.452	78	628.162	773.641	110	81	197

Source: created by the author with information from Resolution Number 1555 of 2010 of the Financial Superintendence of Colombia

Table A2
Mortality table for Mexico

x	l(x) men	l(x) women	x	l(x) men	l(x) women	x	l(x) men	l(x) women
15	977.833	986.289	47	886.534	953.883	79	539.385	812.518
16	976.151	985.381	48	881.490	952.538	80	519.810	796.755
17	974.423	984.475	49	876.219	951.148	81	499.668	778.852
18	972.659	983.569	50	870.707	949.711	82	478.987	758.493
19	970.850	982.654	51	864.952	948.211	83	457.801	735.314
20	968.996	981.740	52	858.932	946.656	84	436.156	708.930
21	967.087	980.827	53	852.636	945.027	85	414.108	678.921
22	965.133	979.915	54	846.053	943.326	86	391.722	644.880
23	963.116	978.994	55	839.167	941.543	87	369.069	606.420
24	961.045	978.074	56	831.958	939.670	88	346.234	563.237
25	958.912	977.145	57	824.421	937.687	89	323.310	515.192
26	956.706	976.216	58	816.531	935.596	90	300.397	462.411
27	954.439	975.279	59	808.276	933.379	91	275.924	405.410
28	952.091	974.343	60	799.635	931.017	92	250.519	345.222

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29	949.673	973.398	61	790.591	928.504	93	224.368	283.517
30	947.166	972.444	62	781.120	925.811	94	197.814	222.624
31	944.570	971.481	63	771.215	922.922	95	171.271	165.374
32	941.888	970.510	64	760.850	919.821	96	145.217	114.743
33	939.109	969.529	65	750.008	916.473	97	120.172	73.248
34	936.226	968.540	66	738.668	912.844	98	96.673	42.275
35	933.240	967.533	67	726.820	908.900	99	75.241	21.627
36	930.141	966.517	68	714.442	904.601	100	56.329	9.597
37	926.923	965.483	69	701.511	899.897	101	40.277	3.609
38	923.577	964.431	70	688.014	894.732	102	27.264	1.123
39	920.095	963.360	71	673.944	889.042	103	17.277	283
40	916.479	962.272	72	659.279	882.756	104	10.102	56
41	912.712	961.155	73	644.010	875.782	105	5.345	9
42	908.788	960.021	74	628.122	868.023	106	2.492	1
43	904.698	958.860	75	611.615	859.360	107	984	0
44	900.437	957.671	76	594.484	849.649	108	309	0
45	895.998	956.445	77	576.732	838.731	109	69	0
46	891.365	955.182	78	558.364	826.427	110	8	0

Source: created by the author with information from Annex 14.2.5_a of the Single Insurance and Bonding Circular