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Valuation of options with adjustments to α-stable distributions and accounting under international financial reporting regulation

Valuación de opciones con ajustes a distribuciones α-estables y contabilidad bajo la norma internacional de información financiera

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Abstract

This paper pretends to analyze the returns of US dollar, euro, sterling and yen, with the Mexican peso, descriptive statistics and α-stable parameters are estimated, goodness of fit tests statistically justify the suitability of α-stable distributions to model the returns of currencies, the self-similarity exponents and memory indices are also estimated, the European call and put option's pricing is done with the Gaussian model and with the α-stable model, and the accounting is presented under international financial reporting standard, concluding that the α-stable model quantify more adequately the exchange rate risk than the Gaussian model, recommending an analysis to minimize the potential losses arising from the economic obligations acquired for issuing options and that international financial reporting standard is aligning the risk management objectives to reflects the risk management activities and transmitting the goal and effect of the options.

JEL Code: C16, C46, C14, D81, G12, G13
Keywords: stochastic processes-stables; finance engineering; international financial reporting standard

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Resumen

En este trabajo se pretende analizar los rendimientos del dólar estadounidense, euro, libra esterlina y yen, con el peso mexicano, son estimados los estadísticos descriptivos y los parámetros α-estables, las pruebas de bondad de ajuste justifican estadísticamente la idoneidad de las distribuciones α-estables para modelar el comportamiento de los rendimientos, también son estimados los exponentes de autosimilitud y los índices de memoria, la valuación de las opciones europeas de compra y de venta es realizada con el modelo gaussiano y con el modelo α-estable, y la contabilización es presentada bajo la norma internacional de información financiera concluyendo que las opciones α-estables cuantifican más adecuadamente el riesgo de tipo de cambio que las opciones gaussianas, recomendando realizar un análisis para minimizar las pérdidas potenciales derivadas de las obligaciones económicas adquiridas por la emisión de opciones y que la norma internacional de información financiera alinea los objetivos de gestión de riesgos para reflejar las actividades transmitiendo el objetivo y el efecto de las opciones.

Código JEL: C16, C46, C14, D81, G12, G13
Palabras clave: procesos estocásticos-estables; ingeniería financiera; normas internacionales de información financiera

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Introduction

Financial engineering uses stochastic α-stable processes to model the returns of financial products that present financial and economic impacts of significant magnitudes due to extreme values and skewness of returns. Financial markets have evolved with information technologies, global competition, financial engineering, and risk management that have innovated the structure of products to meet the needs of investors and issuers. In the context of option valuation, Bachelier (1900) applied a Gaussian stochastic process to model the prices of financial products on the Paris stock exchange. Kendall and Hill (1953) and Kruizenga (1956) conducted an empirical analysis and rejected the ability of the Gaussian stochastic process to model stock prices. Osborne (1959) justified the relevance of the Gaussian stochastic process for modeling returns. Osborne (1959), Sprenkle (1961), Boness (1964), and Samuelson (1965) applied a Gaussian process to value options. Black and Scholes (1973) and Merton (1973) modeled returns as a Gaussian stochastic process and presented a model for valuing options.

The distribution of returns determines the prices of derivative products; then, the valuation of options is obtained by maximizing the present value of the conditional expectation of the contingent payment as a function of the risk-neutral measure, as proposed by Cox and Ross (1976) and Ross (1976). Therefore, the estimation of parameters of the distribution of returns determines the prices of options on financial products.

Merton (1976) proposed a model where returns evolve as stochastic processes with discontinuities where changes are composed of two factors: moderate and independent changes that are modeled with the Gaussian stochastic process and occasional changes of a higher magnitude that are modeled with a compound Poisson stochastic process. The Merton (1976) model is a diffusion process

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with Gaussian jumps, and if the intensity of the jumps is zero, then the Black and Scholes (1973) model is obtained, which is a particular case of the Merton (1976) model. Both models are stochastic Lévy processes (1937).

Sierra Juarez (2007) applied a fractional Gaussian stochastic process to model the evolution of parities and indices. He valued options in fractional markets where the fractional Gaussian stochastic process is a particular case of α-stable stochastic processes. Models for valuing options adjust returns to stochastic processes to ground empirical behavior with probability distributions, and Lévy's stochastic processes have been successful. Esscher (1932) proposed an equivalent probability measure that preserves the properties of Lévy's stochastic processes. The Girsanov transform (1960) for stochastic processes with jumps is a particular case of the Esscher transform. Itô (1942) proposed the Lévy-Itô decomposition, which asserts that stochastic Lévy processes are the sum of a Brownian motion with the trend and a series of compound and independent Poisson processes.

Mandelbrot (1963) proposed a symmetric α-stable process to model cotton prices. Fama (1963, 1965a, 1965b) and Mandelbrot and Taylor (1967) rejected the Gaussian assumption and proposed α-stable stochastic processes. McCulloch (1978, 1985, 1985, 1987, 1996) modeled returns as symmetric α-stable processes and proposed value options with symmetric α-stable distributions. Janicki et al. (1997), Popova and Ritchken (1998), and Hurst, Platen, and Rachev (1999) developed models for valuing options on underlying prices with returns that exhibit symmetric α-stable distributions. Carr and Wu (2003) proposed a finite moments model if the risk-neutral measure has maximum negative skewness and applied α-stable stochastic processes because they retain shape over scale and are suitable for modeling the structure of the implied volatility smile that captures changes in volatility originating from occasional higher magnitude changes across the extremes of the distribution.

McCulloch (2003) formulated a model for valuing options by applying the Esscher transform (1932) as the convolution of a negative extreme α-stable distribution and an exponentially fitted positive extreme α-stable distribution.

In Mexico, Contreras Piedragil and Venegas Martínez (2011), Climent Hernández and Venegas Martínez (2013), Rodríguez Aguilar and Cruz Aké (2013), and Climent Hernández and Cruz Matú (2017) have presented research where options with α-stable distributions are valued, statistically testing the suitability and indicating the differences with the Gaussian model.

The underlying volatility is a factor that significantly influences option valuation and evolves randomly, so it is possible to model it with stochastic processes. For example, Hull and White (1987) modeled volatility as a log-Gaussian process without mean reversion. Scott (1987) modeled volatility as a stochastic process with mean reversion. Stein and Stein (1991) assumed that volatility is uncorrelated with the underlying price and does not capture correlation skewness effects. Heston (1993) proposed a model in which the underlying price is modeled by a stochastic log-Gaussian process, and volatility is modeled with a stochastic Uhlenbeck and Ornstein (1930) process with mean reversion where price and volatility correlate. Heston (1993) derived a formula for valuing European call options. Venegas Martínez (2005) developed a Bayesian model for valuing derivative products with a priori information on volatility, such as expected values, and presented approximate formulas for valuing call options with asymptotic and polynomial approximations of Bessel functions.

The valuation and determination of the optimal early exercise of American options consist of determining the optimal frontier to maximize the cash flow from the early exercise. There are now approximation methods such as the numerical method of Villeneuve and Zanette (2002), the finite difference method of Brennan and Schwartz (1977), and an approximate analytical formula of Barone Adesi and Whaley (1987). Cox et al. (1979) modeled prices as a stochastic binomial process and presented a model for valuing options in discrete time that converges with the Black and Scholes (1973) model with European options and allows the valuation of American put options considering cash flows from anticipated exercise. Climent Hernandez (2014) estimated the risk-free probability parameters and applied stochastic dynamic programming to model the underlying price as a binomial process modeling interest rates and volatility as deterministic functions or as stochastic processes and included dividend payments, showing the difference with the Cox et al. (1979) model and the convergence with the model proposed by Barone Adesi and Whaley (1987).

Martínez Palacios et al. (2012) indicate that the Cox et al. (1979) model and the Climent Hernández (2014) model converge, through a stochastic optimal control approach, to a European option with the Black and Scholes (1973) model plus a positive value satisfying other conditions, and as shown, the value is close to the Barone Adesi and Whaley (1987) model.

In the context of the International Financial Reporting Standard (IFRS), Landeros Olascoaga (2008) indicates that debt or equity financial products with a term of less than one year and listed on the stock market and investments for trading or sale are classified as temporary investments and are presented in current assets. Morales Díaz (2012) analyzes hedge accounting using options, contextualizes hedge accounting regulations, studies in which case a portfolio of options ceases to be a hedging product, discusses the problem of the time value of options and cash flow accounting, and presents the changes introduced by IFRS in hedging with options. Pelmeneva and Talipova (2015) indicate that globalization and integration of companies into the world economy require unification of financial reporting, transparency, uniformity, and procedures of financial calculations because IFRS indicates how financial reports must be generated, imposing obligations in the presentation procedure. Glover and Werner (2015) provide a template for teaching IFRS in accounting curricula, identify instructions for teaching IFRS, present recommendations for improving accounting practices, and indicate that surveys show that IFRS integration in classrooms does not match international accounting expectations and that accounting firms expect candidates to know international accounting standards in order to adopt IFRS. Mantilla (2016) indicates that retrospective valuation of hedge effectiveness is not required; IFRS issued in 2014 replaces previous versions and is mandatory for periods beginning on or after January 1, 2018. Financial statement presentation requires that gains and losses be presented separately in the income statement, and hedge accounting allows risk management activities to be reflected in the financial statements. Gómez Pinto et al. (2019) present the calculations for the accounting of derivative products based on IFRS, indicating that IFRS-9 is to disclose the requirements for the classification and measurement of financial assets and hedge accounting. They also indicate that in the income statement the periodic changes in the valuations of the asset or liability are recorded and that the value of derivative products is classified in the balance sheet as an asset or liability, depending on the rights or obligations established in the contract; in the case of options, the rights or obligations conclude on the maturity date and the transaction is settled.

The objectives of the study are to model the evolution of returns with stochastic α-stable processes to value European call options on the exchange rate parities of the US dollar, euro, pound sterling, and yen with the Mexican peso by estimating the parameters of the distribution of returns with the maximum likelihood method to justify the relevance of the Kolmogorov and Smirnov (KS) and Anderson and Darling (AD) goodness-of-fit tests to manage financial risks more adequately, innovate with the estimation of memory indices and risk functions, and perform the accounting of Gaussian and α-stable options under IFRS.

The paper is organized as follows: section 2 presents an analysis of the performance of the parities, the estimation of the descriptive statistics of the returns, the estimation of the α-stable parameters, the goodness-of-fit tests to the α-stable distributions, a qualitative analysis of the fit of the returns to the Gaussian and α-stable distributions, the estimation of the self-similarity exponents, the memory indices, and the risk functions. In section 3, the valuation of European call and put options is performed. In section 4, European Gaussian and α-stable options are accounted for under IFRS-9. The conclusions are presented in section 5, and finally, the bibliographical references are presented.

Analysis of exchange rate parities

The trajectories of the exchange rate parities of the dollar, euro, pound sterling, and yen during the business period from January 3, 2011, to July 26, 2018, are presented in Figure 1.

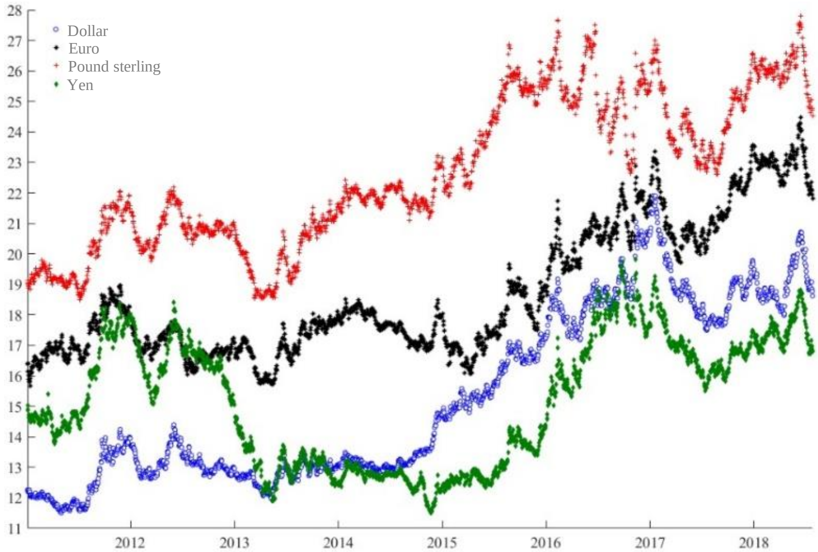

Figure 1. Trajectories of exchange rate parities
Source: created by the authors with data from Banco de México

Figure 1 presents the trajectories of the dollar (circumference), euro (asterisk), pound sterling (cross), and yen multiplied by one hundred pesos (rhombus) with 1 901 daily parities for each exchange rate. Stylized events indicate that the parities show positive skewness and leptokurtosis. Descriptive statistics for the first 1 882 parities are presented in Table 1.

Table 1
Descriptive statistics on exchange rate parities

Parity	Minimum	Maximum	Average	Deviation	Skewness	Kurtosis
Dollar	11.5023	21.9076	15.2903	2.8105	0.4608	1.7124
Euro	15.6622	24.4712	18.6619	2.1480	0.7887	2.3505
Pound	18.5007	27.7979	22.6218	2.4146	0.1667	1.8278
Yen	0.1152	0.1981	0.1518	0.0209	0.0368	1.6083

Source: created by the authors with data from Banco de México

Table 1 presents ranges, averages, standard deviations, skewness, and kurtosis coefficients. The dollar presents the greatest volatility, followed by the pound sterling, the euro, and the yen. The dollar

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presents the highest coefficient of variation[1] (0.1838), followed by the yen (0.1378), the euro (0.1151), and the pound sterling (0.1067), confirming the stylized events identified in Figure 1.

Returns analysis

The 1 881 daily exchange rate returns are presented in Figure 2.

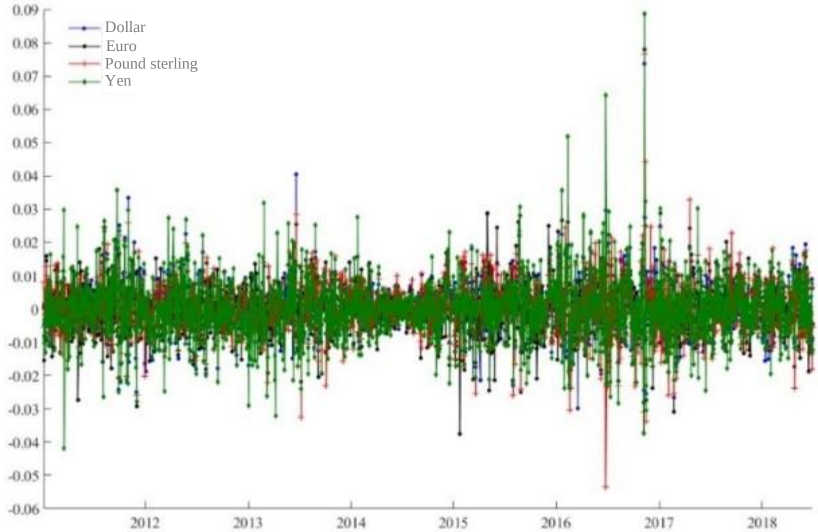

Figure 2. Daily returns on exchange rate parities
Source: created by the authors with data from Banco de México

Figure 2 presents the daily returns of exchange rate parities with 1,881 observations with high volatility clusters representing relevant changes over short periods and moderate volatility clusters representing moderate changes over periods longer than the periods of relevant changes. The estimation of the descriptive statistics of the currency returns is presented in Table 2.

[1] The coefficient of variation of the sample is $v = \frac{S_X}{\bar{X}}$.

Table 2
Descriptive statistics of exchange rate parity returns

Parity	Minimum	Maximum	Average	Deviation	Skewness	Kurtosis
Dollar	-0.029854	0.073724	0.000252	0.007054	0.795047	11.148338
Euro	-0.037420	0.078107	0.000178	0.007742	0.482035	9.463255
Pound	-0.053581	0.076503	0.000165	0.007719	0.299183	10.785422
Yen	-0.041742	0.088755	0.000090	0.009428	0.745728	9.394673

Source: created by the authors with data from Banco de México

Table 2 presents ranges, averages, standard deviations, skewness, and kurtosis coefficients of returns of the parities. The returns show positive averages. Yen returns have the highest standard deviation, followed by the euro, pound sterling, and dollar. The returns have positive skewness coefficients, so the returns have distributions that spread toward positive values more frequently than negative values. The kurtosis coefficients indicate that the distributions are leptokurtic. Therefore, currency returns exhibit asymmetric and leptokurtic distributions compared to the Gaussian distribution. The Gaussian and empirical probabilities of the currency returns are presented in Figure 3.

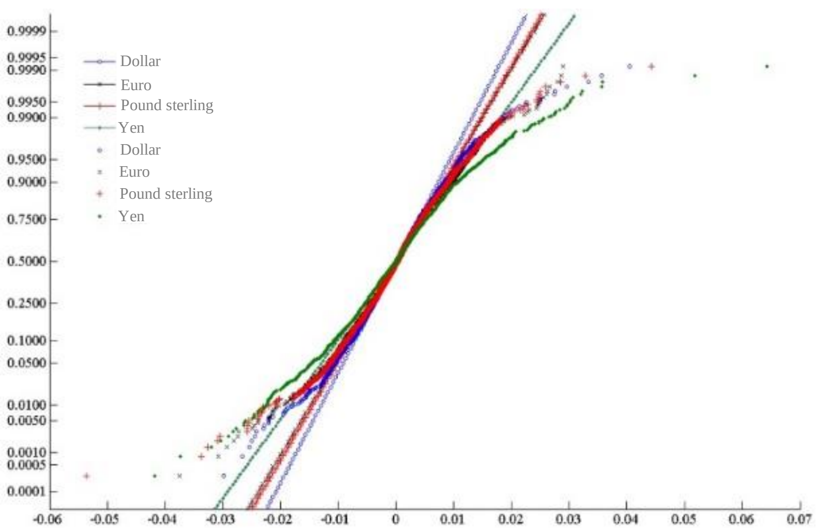

Figure 3. Gaussian probabilities and probabilities of currency returns
Source: created by the authors with data from Banco de México

Figure 3 presents the Gaussian probabilities (straight lines with symbols) as a function of currency returns: dollar (circles), euro (x's), pound sterling (crosses), and yen (diamonds) and the empirical probabilities of currency returns (symbols) as a function of currency returns. They confirm that the returns exhibit skewness and extreme events in the distributions of returns. The quantiles of the Gaussian and empirical returns of the currencies are presented in Figure 4.

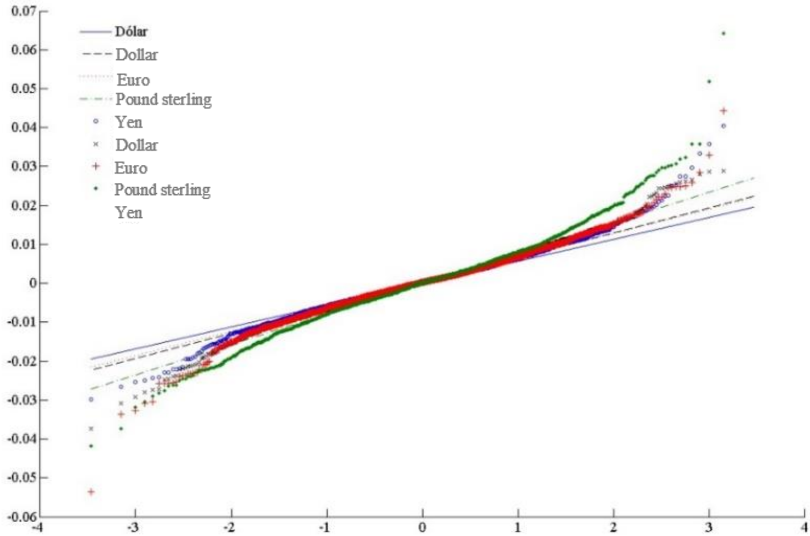

Figure 4. Gaussian quantiles and currency quantiles
Source: created by the authors with data from Banco de México

Figure 4 presents the Gaussian quantiles (lines) as a function of the Gaussian quantiles of the currencies: dollar (solid line), euro (dashed line), pound sterling (dotted line), and yen (dashed and dotted line) and the quantiles of the currencies (symbols) as a function of the Gaussian quantiles of the currencies: dollar (circles), euro (x's), pound sterling (crosses), and yen (diamonds). Confirming that the returns exhibit skewness and leptokurtosis in the returns distributions, the Gaussian distributions are underestimating relevant losses and gains. Therefore, investors who do not consider skewness and leptokurtosis under the Gaussian assumption are exposed to risks they are not considering.

Qualitative analysis indicates that the returns do not exhibit Gaussian distributions; then, the returns are fitted to α-stable distributions to model skewness and leptokurtosis characteristics to more adequately quantify relevant events that are inadequately accounted for by the Gaussian distribution and represent risks to investors due to the lack of a model consistent with empirical currency returns.

Estimation of α-stable parameters

Four parameters characterize the α-stable distributions and are generally denoted by $S(\alpha, \beta, \gamma, \delta)$. The stability parameter $0 < \alpha \leq 2$ determines the degree of leptokurtosis and the slope with which the extremes of the distribution decrease. The skewness parameter $-1 \leq \beta \leq 1$ defines the degree of skewness of the distribution. The scale parameter $\gamma > 0$ indicates the units of dispersion that the distribution has relative to the location parameter. The location parameter $-\infty < \delta < \infty$ determines the location point that the distribution has. The α-stable distributions generally do not have a closed analytical expression to characterize the random variable, but with the characteristic function $\varphi_Z(\kappa)$ or with the cumulant function, $\psi_Z(\kappa)$ it is possible to characterize any α-stable random variable uniquely. A random variable Y is α-stable if and only if $Y = \gamma Z + \delta$, where Z is a random variable with the characteristic function:

$$\varphi_z(\kappa) = E(\exp(\imath\kappa Z)) = \begin{cases} \exp\left(-|\kappa|^\alpha \left(1 - \imath\beta\operatorname{sgn}(\kappa)\tan\left(\frac{\pi\alpha}{2}\right)\right)\right) & \text{si } \alpha \neq 1, \\ \exp\left(-|\kappa|\left(1 - \imath\frac{2}{\pi}\beta\operatorname{sgn}(\kappa)\ln(\kappa)\right)\right) & \text{si } \alpha = 1, \end{cases} \tag{1}$$

where:

$$\operatorname{sgn}(\kappa) = \begin{cases} \frac{\kappa}{|\kappa|} & \kappa \neq 0, \\ 0 & \kappa = 0. \end{cases}$$

The cumulant function of the random variable Z is:

$$\psi_Z(\kappa) = \ln(\varphi_Z(\kappa)) = \begin{cases} -|\kappa|^\alpha \left(1 - \imath\beta\operatorname{sgn}(\kappa)\tan(\theta)\right) & \text{si } \alpha \neq 1, \\ -|\kappa|\left(1 - \frac{2\imath}{\pi}\beta\operatorname{sgn}(\kappa)\ln(\kappa)\right) & \text{si } \alpha = 1, \end{cases} \tag{2}$$

where $\iota^2 = -1$. The α-stable distributions have closed analytic expressions for the following cases: Gauss: $S\left(2,0,2^{-1}\sigma,\mu\right)$, Cauchy: $S\left(1,0,\gamma,\delta\right)$, and Lévy: $S\left(2^{-1},\pm 1,\gamma,\delta\right)$.

The returns analysis indicates that the distributions are skewed and leptokurtic, so the estimation of α-stable parameters is performed with the maximum likelihood method and parameterization 1. The estimation of the α-stable parameters is presented in Table 3.

Table 3
Estimation of α-stable parameters at 95% confidence

Parity	α	β	γ	δ
Dollar	1.7793±0.0639	0.1436±0.2386	0.004195±0.000173	0.000257±0.000331
Euro	1.8259±0.0601	0.0000±0.2968	0.004809±0.000193	0.000152±0.000376
Pound	1.7832±0.0641	0.0000±0.2467	0.004587±0.000190	0.000186±0.000362
Yen	1.7862±0.0634	0.1605±0.2421	0.005694±0.000234	0.000066±0.000449

Source: created by the authors with data from Banco de México

Table 3 presents the estimations of the α-stable parameters. The stability and skewness parameters are consistent with the estimates presented internationally by Dostoglou and Rachev (1999), Ortobelli et al. (2002), Ortobelli et al. (2004), Rachev et al. (2004), Čížek et al. (2005), Ortobelli et al. (2005), Scalas and Kim (2006), and nationally by Contreras Piedragil and Venegas Martínez (2011), Climent Hernández and Venegas Martínez (2013), Climent Hernández and Cruz Matú (2017), and Climent Hernández et al. (2017). The scale parameters indicate that the yen has the highest dispersion, followed by the euro, the pound, and the dollar. Thus, dollar and yen returns exhibit positive skewness and leptokurtosis, and euro and sterling returns exhibit only leptokurtosis.

Kolmogorov and Smirnov goodness-of-fit test

The analysis to test the hypothesis that the returns present Gaussian or α-stable distributions with the goodness-of-fit statistic KS is presented in Table 4.

Table 4
Gaussian KS tests at 99% confidence

Parity	D	$P\left(D>d\right)$	D_α	$P_\alpha\left(D>d\right)$
Dollar	0.0552	0.0000	0.0195	0.4675
Euro	0.0456	0.0008	0.0187	0.5256
Pound	0.0475	0.0004	0.0196	0.4632
Yen	0.0496	0.0002	0.0271	0.1231

Source: created by the authors with data from Banco de México

Table 4 presents the D statistics for the currency returns and descriptive significance levels, rejecting the hypotheses that the returns exhibit Gaussian distributions and not rejecting the hypotheses that the returns exhibit α-stable distributions. The KS tests indicate that empirical returns exhibit extreme events and that α-stable distributions model skewness and leptokurtosis more adequately than the Gaussian distribution, allowing market risks to be quantified more appropriately. The KS goodness-of-fit tests are supplemented with AD statistics to justify the Gaussian hypotheses' relevance and fit to α-stable distributions.

Anderson and Darling's goodness-of-fit test

The quantitative analyses to test the hypothesis that the returns present Gaussian or α-stable distributions are performed with the goodness-of-fit statistic AD presented in Table 5.

Table 5
Gaussian AD tests at 99% confidence

Parity	A^2	$P\left(A^2 > a^2\right)$	A_{α}^2	$P_{\alpha}\left(D > d\right)$
Dollar	∞	0.0000	0.8210	0.4661
Euro	∞	0.0000	0.6751	0.5799
Pound	∞	0.0000	0.6889	0.5682
Yen	49.0196	0.0000	0.5891	0.6583

Source: created by the authors with data from Banco de México

Table 5 presents the A^2 statistics for the currency returns and descriptive significance levels, rejecting the hypotheses that the returns exhibit Gaussian distributions and not rejecting the hypotheses that the returns exhibit α-stable distributions. The AD tests indicate that empirical returns exhibit extreme events that α-stable distributions model more adequately than the Gaussian distribution, allowing market risks to be managed more effectively for decision-making.

The results in Tables 4 and 5 indicate that α-stable distributions are more efficient in modeling the empirical behavior of returns and managing exchange rate risks effectively than the Gaussian distribution. Therefore, and according to the KS and AD goodness-of-fit tests, the α-stable distributions are suitable for modeling parity returns in the period studied, and it is possible to value options based on exchange rates.

Adjustments of distributions

The fits of the Gaussian and α-stable distributions to the dollar, euro, pound sterling, and yen returns are presented in Figure 5.

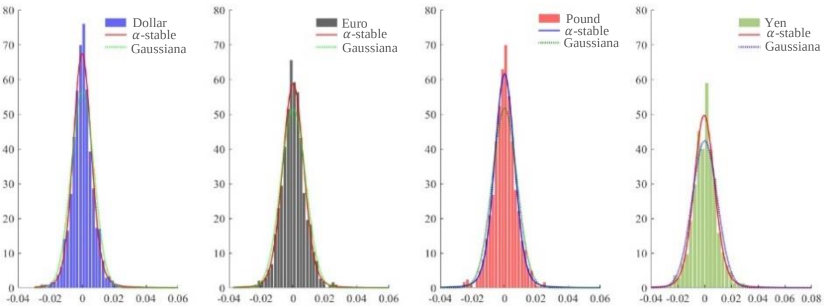

Figure 5. Adjustment of the α-stable distribution to currency returns
Source: created by the authors with data from Banco de México

Figure 5 presents the fits of the Gaussian and α-stable distributions to the currency returns, confirming that α-stable distributions more adequately model the properties of skewness and leptokurtosis than Gaussian distributions. Therefore, the goodness-of-fit tests and the fits presented in Figure 5 show that α-stable distributions are efficient for modeling the behavior of returns and quantifying exchange rate risks.

Estimation of self-similarity exponents

The process $X(t)$ is self-similar with exponent $H>0$, if for all $a\in(0,\infty)$, the finite-dimensional distributions of $X(at)$ are identical to the finite-dimensional distributions of $a^H X(t)$:

$$\left(X(at_1),\ldots,X(at_n)\right)\underline{\underline{d}}\left(a^H X(t_1)\ldots,a^H X(t_n)\right)$$

(3)

Lévy's symmetric α-stable motion (MES) is self-similar with $H=\alpha^{-1}$, therefore, the self-similarity exponent $H\in\left[2^{-1},\infty\right)$, i.e., MB is self-similar with $H=2^{-1}$.

Belov et al. (2006) indicate that α-stable processes are a powerful and versatile tool for financial models. They demonstrate the efficiency of α-stable parameters estimated by the maximum likelihood method. They perform hypothesis tests for self-similarity and multifractality. They estimate the Hurst exponent in the time domain with the absolute moments (AM), variance convergence (CV), rescaled rank (RR), and variance of residuals (VR) methods, and in the frequency domain, they use the periodogram (PG) and Whittle and Abry Veitch (WAV) methods.

Barunik and Kristoufek (2010) show that the properties in the estimation of the Hurst exponent change with the presence of leptokurtosis. They perform Monte Carlo simulations to analyze RR, fluctuation analysis without multifractional trends (AFSTMF), trendless moving mean (MMST), and generalized Hurst exponent (GHE). They estimate the Hurst exponent from independent series with different stability parameters. They indicate that the GHE method provides the lowest variance and the lowest skewness compared to other methods. They also indicate that GHE is $H(q) \approx q^{-1}$ for $q > \alpha$ and that $H(q) \approx \alpha^{-1}$ for $q \leq \alpha$. They indicate that GHE is suitable for multifractional screening and is comparable to RR, MMST and AFSTMF(2). If q=1, then H(1) characterizes the scale of the absolute deviations of the process. The GHE(1) and AFSTMF(1) methods present $\mathrm{E}(H) = \alpha^{-1}$, therefore, GHE(1) presents the best performance for finite samples among all methods, with the lowest variance, the smallest bias, and the narrowest confidence intervals. They conclude that RR and GHE are robust; GHE (q) outperforms all the methods mentioned. Therefore, GHE(q) is useful because it presents satisfactory properties outperforming the other methods.

Climent Hernández et al. (2017) estimate the ordered pair (α, H) to find the α-stable distribution forms, the factional dimensions of the probability spaces $(\Omega, \mathcal{F}, P)$, the fractional dimensions of the time series, the anti-persistence, stochastic independence, or persistence effects, and the stochastic processes with which it is possible to properly model the time series of the dollar, euro, yen, and Canadian dollar parities employing GHE(1) to estimate the H self-similarity exponents, and perform the t and F tests, ruling out that the parity series are multifractional.

The estimation of the self-similarity exponent proposed by Climent Hernández et al. (2017) is performed by the GHE(1) method, where $\mathrm{E}(H) = \alpha^{-1}$ is the boundary between anti-persistence (reversion to the mean) and persistence (long memory) for a-stable processes. The method consists of performing 19 partitions $K_q(\tau)$ of the returns, where $\tau = 1, \ldots, 19$, then:

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$$K_q(\tau) = \frac{\sum_{t=1}^{\nu} \left| X(t+\tau) - X(t) \right|^q}{\sum_{t=1}^{\nu} \left| X(t) \right|^q} \tag{4}$$

where $K_q(\tau)$ is the scale statistic, $q = 1, \ldots, 10$ is the moment, τ is the increment between cumulative returns, n is the size of the time series, $\nu = \lceil \tau^{-1} n \rceil$ is the size of the partition, $X(t)$ is the cumulative return at time t, and $\lceil \cdot \rceil$ is the upper integer function, then, $K_q(\tau) \approx c\tau^{qH(q)}$. With the 19 data the linear regression is calculated and the first estimator GHE(q) is obtained, then 15 regressions are performed for $\tau = 5, \ldots, 19$, and by calculating the arithmetic average $\mathrm{EHG}(1)$ is obtained. For the multifractal analysis, the first ten moments are considered $q = 1, \ldots, 10$, and the regression analysis is performed. If it is linear, then the series is self-similar. Otherwise, the series is multifractional.

The self-similar processes are distribution invariant under time and space scales; the estimates of the self-similarity exponents, the memory indices, the fractional dimensions of the time series, and the hazard functions for the returns are presented in Table 6:

Table 6
Estimation of self-similarity exponents

Parity	$\mathrm{EHG}(1)$	Minimum	Maximum	s_H	$\aleph$	D	$\square(\aleph_k)$
Dollar	0.5141	0.5024	0.5268	0.0074	0.9148	1.4859	1.0852
Euro	0.5102	0.4926	0.5227	0.0099	0.9317	1.4898	1.0683
Pound	0.5201	0.5076	0.5270	0.0048	0.9275	1.4799	1.0725
Yen	0.5176	0.5062	0.5287	0.0070	0.9245	1.4824	1.0755

Source: created by the authors with data from Banco de México

Table 6 presents the self-similarity exponents, ranges, and standard deviation. Memory indices $\aleph = \alpha H$ indicate that the returns are reverting to the mean. The indices $D = 2 - H$ indicate that the dimensions of the time series are fractional. The risk functions $\square = 2 - \aleph$ indicate that the risk measures are underestimated because they ignore the probabilities of change in the trend of returns. The results are consistent with Climent Hernández et al. (2017), Climent Hernández and Aguilar Vázquez

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(2017), and Climent Hernández and Rodríguez Benavides (2018). The results of the fifteen regressions to estimate the self-similarity exponents are shown in Table 7.

Table 7
ANOVA of self-similarity exponents

Parity	R^2	$P\left(T>t\right)$	$P\left(F>f\right)$
Dollar	[0.9937,0.9973]	[1.9E-20,1.6E-04]	[1.9E-20,1.6E-04]
Euro	[0.9887,0.9969]	[5.4E-18,7.7E-05]	[5.4E-18,7.7E-05]
Pound	[0.9949,0.9999]	[6.5E-21,4.1E-07]	[6.5E-21,4.1E-07]
Yen	[0.9938,0.9964]	[1.4E-20,1.9E-04]	[1.4E-20,1.9E-04]

Source: created by the authors with data from Banco de México

Table 7 shows the ranges of the coefficients of determination, indicating that the self-similarity exponents have a goodness-of-fit greater than 98.87%, the self-similarity exponents are different from zero, and the model is linear, so the self-similarity exponents estimated with the GHE(1) method are non-significant and, therefore, the parities are self-similar. The linearity of the GHE(q) method for the moments $q=1,\ldots,10$ determines whether the parities are self-similar or multifractional. The model estimates are presented in Table 8.

Table 8
Simple linear regression model

Parity	$\mathrm{EHG}\left(q\right)$	R^2	$P\left(T>t\right)$	$P\left(F>f\right)$
Dollar	-0.0390	0.9726	1.55E-07	1.55E-07
Euro	-0.0420	0.9765	8.36E-08	8.36E-08
Pound	-0.0294	0.9753	1.02E-07	1.02E-07
Yen	-0.0391	0.9781	6.39E-08	6.39E-08

Source: created by the authors with data from Banco de México

Table 8 presents the slopes. The coefficients of the determination indicate that the self-similarity exponents have a goodness of fit greater than 97.26%. The slopes are non-zero, and the model is linear, so the currency parities are self-similar. Therefore, statistical analysis indicates that parity returns are self-similar, and applying α-stable stochastic processes for the valuation of options on dollar, euro, pound sterling, and yen exchange rate parities is relevant.

Valuation of European options

Options are a contingent right that depends on the underlying price at the maturity date. The valuation of European options through the Gaussian model is:

$$\begin{aligned} c(t,M_t) &= M_t \exp(-r\tau)\Phi(d_1) - S\exp(-i\tau)\Phi(d_2) \\ p(t,M_t) &= S\exp(-i\tau)\Phi(-d_2) - M_t \exp(-r\tau)\Phi(-d_1) \end{aligned} \tag{5}$$

where:

$$d_1 = \frac{\ln\left(\frac{M_t}{S}\right) + \left(i - r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \quad \text{y} \quad d_2 = d_1 - \sigma\sqrt{\tau} \tag{6}$$

where:

$$\begin{aligned} c(t,M_t) &= \begin{cases} M_t \exp(-r\tau)\Phi(d;\alpha,\beta) - S\exp(-i\tau)\Phi(d;\alpha,-\beta) \\ M_t \exp(-r\tau)\big(1-\Phi(-d;\alpha,-\beta)\big) - S\exp(-i\tau)\big(1-\Phi(-d;\alpha,\beta)\big) \end{cases} \\ p(t,M_t) &= \begin{cases} S\exp(-i\tau)\Phi(-d;\alpha,\beta) - M_t \exp(-r\tau)\Phi(-d;\alpha,-\beta) \\ S\exp(-i\tau)\big(1-\Phi(d;\alpha,-\beta)\big) - M_t \exp(-r\tau)\big(1-\Phi(d;\alpha,\beta)\big) \end{cases} \end{aligned} \tag{7}$$

where:

$$d = \frac{\ln\left(\frac{M_t}{S}\right) + \left(i - r - \beta\gamma^{\alpha}\sec(\theta)\right)\tau}{\gamma\tau^{\frac{1}{\alpha}}} \quad \text{y} \quad \theta = \frac{\alpha\pi}{2} \tag{8}$$

The α-stable model applied is like the model of Climent Hernández and Cruz Matú (2017), where α, β and γ are the parameters of the α-stable distributions. The valuation of currency options depends on exogenous factors such as parities, volatilities, interest rates, and endogenous factors such as the settlement price and the hedging period. Parities were obtained from banxico.org.mx. Estimated volatilities are historical. Interest rates were obtained from banxico.org.mx, homefinance.nl, and http://es.global-rates.com. Settlement prices were estimated as the forward prices, and the hedging period is July 2, 2018, with maturity on July 26, 2018, which is 24 calendar days or 19 business days. Forward prices are:

$$F_t = M_t \exp((i-r)\tau) \tag{9}$$

The prices of the currencies at the time of trading and the settlement prices at maturity of the options on the exchange rate parities are presented in Table 9:

Table 9
Settlement prices at maturity

Parity	M_0	S
Dollar	19.6912	19.7625
Euro	22.9215	23.0401
Pound	25.8733	25.9918
Yen	0.1780	0.1789

Source: created by the authors with data from Banco de México and Home Finance

Table 9 presents the currency prices at the time of option trading and the settlement prices at maturity for options on the parities calculated with the Equation **¡Error! No se encuentra el origen de la referencia.**. The limits for the valuation of European options are as follows:

$$\begin{gathered} \text{máx}\left(M_t \exp\left(-r\tau\right) - S\exp\left(-i\tau\right), 0\right) \le c\left(t, M_t\right) \le M_t \\ \text{máx}\left(S\exp\left(-i\tau\right) - M_t \exp\left(-r\tau\right), 0\right) \le p\left(t, M_t\right) \le S\exp\left(-i\tau\right) \end{gathered} \tag{10}$$

Option prices satisfy the Equation **¡Error! No se encuentra el origen de la referencia.** to avoid arbitrage opportunities. The valuation of the European Gaussian and α-stable call options on the exchange rate parities for the term are presented in Figure 6.

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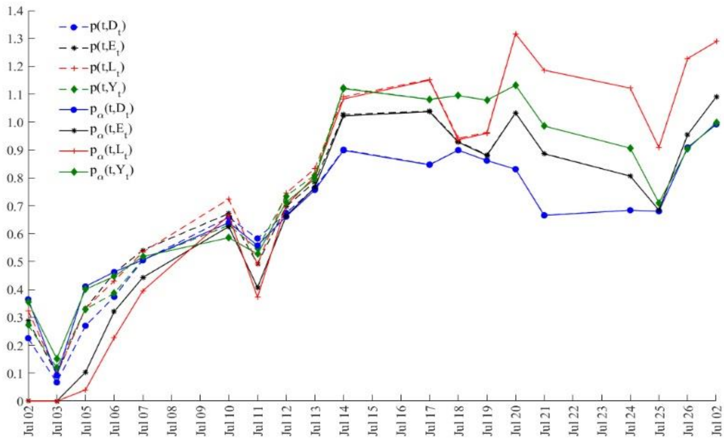

Figure 6. Valuation of call options
Source: created by the authors with data from Banco de México and Home Finance

Figure 6 presents the valuations of Gaussian call options (dashed lines) and the valuations of α-stable call options (solid lines) on the dollar, euro, pound sterling, and yen parities[2]. Gaussian call options on the euro and sterling have a higher value than α-stable call options at the time of issue, and all call options are out-of-the-money at the maturity date. The valuation of the Gaussian and α-stable put options on the parities for the term is presented in Figure 7.

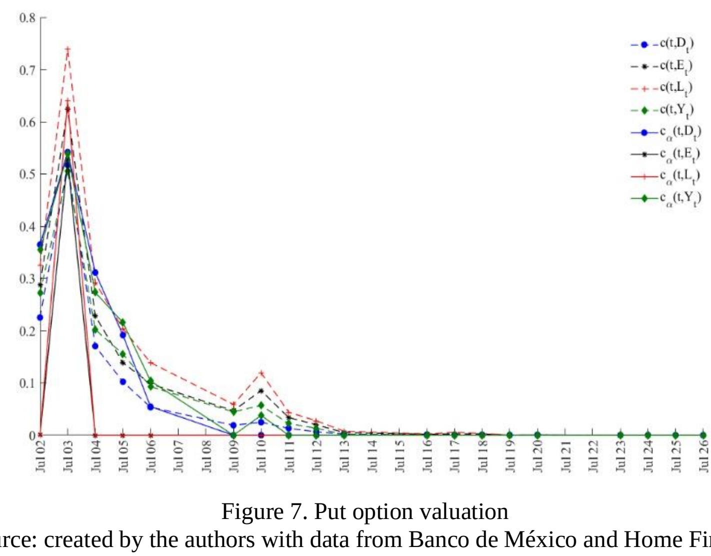

Figure 7. Put option valuation
Source: created by the authors with data from Banco de México and Home Finance

[2] The valuations of European call options on the yen are multiplied by one hundred pesos.

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Figure 7 presents the valuations of Gaussian put options (dashed lines) and the valuations of α-stable put options (solid lines) on the exchange rate parities of the dollar, euro, pound sterling, and yen[3]. Gaussian put options on the euro and pound sterling have a higher value than α-stable call options at the time of issue, and all call options are in-the-money at the maturity date.

Figures 6 and 7 indicate that Gaussian options on the dollar and the yen are less expensive than α-stable options, Gaussian options on the euro and the pound sterling are more expensive than α-stable options, call options are out-of-the-money at maturity, and put options are in-the-money at maturity.

International financial reporting regulations

Rodríguez Díaz (2017) indicates that the International Accounting Standards Board (IASB) promulgated in July 2014 the hedge accounting standard in a chapter of IFRS-9, replacing IAS39, which regulated derivative accounting, and is mandatory as of January 1, 2018.

From an accounting point of view, there are two possibilities when negotiating a derivative product:

1. Record the derivative product as trading or speculative at fair value.
2. Register the derivative as a hedging product.

IFRS-9 aligns risk management objectives with accounting by simplifying hedge efficiency requirements and introducing disclosure requirements for risk management activities so that hedge accounting conveys the purpose and effect of hedging products (derivatives) on an optional basis. For IAS39, hedging products are derivative products, except that non-derivative financial product are used to hedge foreign exchange risks, and IFRS-9 does not govern compliance with hedge accounting when the product is or is not a derivative. The main differences regarding IAS39 are as follows:

1. Global exposures that include a derivative with specific circumstances in which they are hedging net positions can be allocated in IFRS-9, and IAS39 prohibits it.
2. The accounting of the extrinsic value of options in hedging, cash flow, or fair value relations, whose change in fair value can be deferred under certain rules as a hedging cost, is modified in IFRS-9 and IAS39; this component was taken to profit or loss as an inefficiency.
3. The efficiency assessment is aligned with risk management with the economic ratio principle in IFRS-9 and IAS39 with the quantitative prospective and retrospective hedging rule of 80 to 125%. Retrospective evaluation is no longer a requirement but is used to record inefficiency.

[3] The valuation of European put options on the yen is multiplied by one hundred pesos.

4. The fair value option is a solution for cases where it is impossible to apply hedge accounting.

In IAS39, when recording the derivative product as a hedging product, special hedge accounting regulations are applied to minimize accounting skewnesses that arise because the derivative product is offsetting another position that is not measured at fair value through profit or loss, or that does not appear on the balance sheet.

In IFRS-9, the extrinsic value is deferred as a cost for the hedge, so the option hedge is attractive again. The IAS39 hedge accounting model is complex and strict and does not reflect the economic reality of risk management policies in the business model context. IFRS-9 aligns risk management objectives with accounting to reflect risk management activities by conveying the objective and effect of hedging products on an optional basis. IFRS-9 for derivative products indicates the requirements for classifying and measuring financial products and hedge accounting. The cost of derivative products is classified in the balance sheet as an asset or a liability, depending on the rights or obligations established in the contract. The income statement presents the periodic changes in the valuation of financial assets or liabilities.

In Mexico, accounting is governed by the financial reporting regulation (FRS C2), which establishes that derivative products that constitute a temporary investment with a term of less than one year that are quoted in the stock market and investment or hedging products are classified as temporary investments and are presented in current assets or short-term liabilities as appropriate to the rights and obligations.

Income statement

Changes in the value of the underlying price generate changes in the value of the options. Therefore, the short position records premiums collected as gains and contingent payments as losses, and the long position records contingent payments as gains and premiums paid as losses.

Balance sheet

The options are presented as assets or liabilities, respectively; the options with the long position, which grant the right but not the obligation, are presented as assets, in banks as the premium paid; and the options with the short position, which grant the obligation, are presented as assets, in banks as the premium collected and in liabilities as the contingent payment.

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Accounting for European call and put options

Assuming the options cover ten thousand currencies, the issue or purchase for hedging is for one hundred lots covering one hundred currencies each. The T-accounts of the long position of call and put options on currencies are presented in Table 10.

Table 10
T-accounts of the long position of call and put options

	Derivative products		Banks	
	Charge	Payment	Charge	Payment
$v(t,D_t)$		2 252.79	2 252.79	
$v(t,E_t)$		2 882.52	2 882.52	
$v(t,L_t)$		3 241.96	3 241.96	
$v(t,Y_t)$		27.25	27.25	
		8 404.53	8 404.53	
	Derivative products		Banks	
	Charge	Payment	Charge	Payment
$v_\alpha(t,D_t)$		3 647.06	3 647.06	
$v_\alpha(t,E_t)$		1.00	1.00	
$v_\alpha(t,L_t)$		2.00	2.00	
$v_\alpha(t,Y_t)$		35.50	35.50	
		3 685.57	3 685.57	

Source: created by the authors

Table 10 presents the T-accounts for the long position of call and put options on currencies. Premiums paid amounted to \$8 404.53 and \$3 608.57, respectively. Gaussian options on the US dollar and the yen are less expensive than α-stable options. Gaussian options on the euro and the pound sterling are more expensive than α-stable options. Call options are out-of-the-money at maturity, and put options are in-the-money at maturity. Then, Gaussian holders overestimate the value of the call or put option portfolio, and the premium for hedging is higher than that of α-stable holders. The holders acquire rights with the writers, i.e., they acquire contingent gains. The T-accounts of the short position of call and put options on currencies are presented in Table 11.

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Table 11
T-accounts of short position of call or put options

	Derivative products		Banks	
	Charge	Payment	Charge	Payment
$v(t,D_t)$	2 252.79			2 252.79
$v(t,E_t)$	2 882.52			2 882.52
$v(t,L_t)$	3 241.96			3 241.96
$v(t,Y_t)$	27.25			27.25
	8 404.53			8 404.53

	Derivative products		Banks	
	Charge	Payment	Charge	Payment
$v_\alpha(t,D_t)$	3 647.06			3 647.06
$v_\alpha(t,E_t)$	1.00			1.00
$v_\alpha(t,L_t)$	2.00			2.00
$v_\alpha(t,Y_t)$	35.50			35.50
	3 685.57			3 685.57

Source: created by the authors

Table 11 presents the T-accounts for the short position of call and put options on currencies and shows similar behavior to the long position, but the writers of the European call and put options acquire obligations with the holders, i.e., they acquire contingent losses.

Changes in the value of the options are recorded in the income statement. Following MFRS B3, the comprehensive financing result (CFR) comprises revenues and expenditures related to financing activities. The CFR presents the value of financial assets or liabilities, gains, or losses from valuations. The comprehensive financing result of the long position financing of the European call options as part of the income statement as of July 31, 2018, is presented in Table 12.

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Table 12
Income statement of long call option position as of July 31, 2018

	Result of financing	
$\Pi_{c}(t,M_t)$	Gain on hedging	0.00
$\Pi_{c}(t,M_t)$	Loss on hedging	8 404.53
		-8 404.53
	Result of financing	
$\Pi_{c_\alpha}(t,M_t)$	Gain on hedging	0.00
$\Pi_{c_\alpha}(t,M_t)$	Loss on hedging	3 685.57
		-3 685.57

Source: created by the authors

Table 12 presents the result of financing the long position of the European call options as part of the income statement as of July 31, 2018. The loss on the hedge with the Gaussian portfolio ($8 404.53) is greater than the loss on the α-stable portfolio ($3 685.57), and the difference between the gain on the hedge and the premium paid is reported under banks on the balance sheet. The result of financing the long European put option position as part of the income statement as of July 31, 2018, is presented in Table 13.

Table 13
Income statement of long put option position results as of July 31, 2018.

	Result of financing	
$\Pi_{c}(t,M_t)$	Gain on hedging	33 803.59
$\Pi_{c}(t,M_t)$	Loss on hedging	8 404.53
		25 399.06
	Result of financing	
$\Pi_{c_\alpha}(t,M_t)$	Gain on hedging	33 803.59
$\Pi_{c_\alpha}(t,M_t)$	Loss on hedging	3 685.57
		30 118.02

Source: created by the authors

Table 13 presents the result of financing the long European put option position as part of the income statement as of July 31, 2018. The net gain from hedging with the Gaussian portfolio ($25 399.06) is less than that from hedging with the α-stable portfolio ($30 118.02). Therefore, the net gain of the

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Gaussian portfolio is less than the net gain of the α-stable portfolio, and the difference between the gain on the hedge and the premium paid is reported under banks in the balance sheet. The comprehensive financing result of the short position financing of the European call options as part of the income statement as of July 31, 2018, is presented in Table 14.

Table 14
Income statement for the short call option position as of July 31, 2018

	Result of financing	
$\Pi_{c(t,M_t)}$	Gain on hedging	8 404.53
$\Pi_{c(t,M_t)}$	Loss on hedging	0.00
		8 404.53
	Result of financing	
$\Pi_{c_\alpha(t,M_t)}$	Gain on hedging	3 685.57
$\Pi_{c_\alpha(t,M_t)}$	Loss on hedging	0.00
		3 685.57

Source: created by the authors

Table 14 presents the result of financing the short position of the European call options as part of the results as of July 31, 2018. The gain from issuing the Gaussian portfolio hedge is greater than that from issuing the hedge with the α-stable portfolio. The gain on the Gaussian portfolio is greater than the gain on the α-stable portfolio, and the difference between the gain and loss on the hedge is reported under banks on the balance sheet. The comprehensive financing result of the short position financing of the European put options as part of the income statement as of July 31, 2018, is presented in Table 15.

Table 15
Income statement of short put option short position results as of July 31, 2018.

	Result of financing	
$\Pi_{c(t,M_t)}$	Gain on hedging	8 404.53
$\Pi_{c(t,M_t)}$	Loss on hedging	33 803.59
		-25 399.06
	Result of financing	
$\Pi_{c_\alpha(t,M_t)}$	Gain on hedging	3 685.57
$\Pi_{c_\alpha(t,M_t)}$	Loss on hedging	33 803.59
		-30 118.02

Source: created by the authors

Table 15 presents the result of financing the short European put option position as part of the income statement as of July 31, 2018. The net loss from the issuance of the hedge with the Gaussian portfolio ($25 399.06) is less than the net loss from the issuance of the hedge with the α-stable portfolio ($30 118.02). Therefore, the net loss of the Gaussian portfolio is less than the net loss of the α-stable portfolio, and the difference between the hedge gain and the hedge loss is reported under banks in the balance sheet. Therefore, the loss of the Gaussian portfolio is smaller than that of the α-stable portfolio.

IFRS-9 and FRS C2 indicate that the accounting record is presented with the value of the options in the balance sheet. Losses from the long call option position as part of the balance sheet are presented in Table 16.

Table 16
Balance sheet of long call option position as of July 31, 2018

$\Pi_{c(t,M_t)}$	Banks	-8 404.53
	Derivative products	-8 404.53
$\Pi_{c_\alpha(t,M_t)}$	Banks	-3 685.57
	Derivative products	-3 685.57

Source: created by the authors

Table 16 presents the total premiums paid for call options on foreign currencies as an expense for the hedge. The gains from the long-put option position as part of the balance sheet are presented in Table 17.

Table 17
Balance sheet of the long-put option position as of July 31, 2018

$\Pi_{p(t,M_t)}$	Banks	25 399.06
	Derivative products	25 399.06
0 $\Pi_{p_\alpha(t,M_t)}$	Banks	30 118.02
	Derivative products	30 118.02

Source: created by the authors

Table 17 presents the difference between the total proceeds from the settlement payments at maturity of the put options and the total premiums paid for the put options on the foreign currency as income because the settlement payment at maturity is greater than the premiums paid. Earnings from the short call option position as part of the balance sheet are presented in Table 18.

Table 18
Balance sheet of the short call option position as of July 31, 2018

$\Pi_{c(t,M_t)}$	Banks	8 404.53
	Derivative products	8 404.53
$\Pi_{c_\alpha(t,M_t)}$	Banks	3 685.57
	Derivative products	3 685.57

Source: created by the authors

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Table 18 presents the total premiums charged for call options on foreign currencies as income from the hedge granted. Losses from the short put option position as part of the balance sheet are presented in Table 19.

Table 19
Balance sheet of the short put option position as of July 31, 2018

$\Pi_{p(t,M_t)}$	Banks	-25 399.06
	Derivative products	-25 399.06
$\Pi_{p_\alpha(t,M_t)}$	Banks	-30 118.02
	Derivative products	-30 118.02

Source: created by the authors

Table 19 presents the difference between the total premiums collected on foreign currency put options and the total losses from settlement payments at maturity of the put options as an outflow because the settlement payment at maturity is greater than the total premiums collected.

Assuming that the options issue is for ten thousand currencies and only one single position is held, i.e., one hundred lots of options are held or issued on only one of the currencies, then the long and short position on European call and put options present the following results:

The long position of the Gaussian call options on the dollar and yen shows smaller losses than the α-stable call options, and the Gaussian call options on the euro and the pound sterling show larger losses than the α-stable call options.

The long position of the Gaussian put options on the dollar and the yen shows higher gains than the α-stable put options, and the Gaussian put options on the euro and the pound sterling and show lower gains than the α-stable call options.

The short position of the Gaussian call options on the dollar and yen shows lower gains than the α-stable call options—and the Gaussian call options on the euro and the pound sterling and show higher gains than the α-stable call options.

The short position of the Gaussian call options on the dollar and yen shows larger losses than the α-stable call options and the Gaussian call options on the euro and the pound sterling and show lower losses than the α-stable call options.

The option portfolios, with long and short positions, that were analyzed are strategies for hedging and combinations (European call and put options of the same series):

1. Cone long straddle position. Investors expect relevant changes in underlying prices and create the following portfolio:

1.1. The long position of a European call option portfolio.

1.2. The long position of a portfolio of European put options of the same series.

The long cone is also known as the bottom straddle. It generates limited losses if the underlying portfolio is priced close to settlement prices, limited gains if the underlying portfolio is priced below settlement prices, and unlimited gains if the underlying portfolio is priced above settlement prices, i.e., investors expect underlying prices to be far from settlement prices.

2. Cone short straddle position. Investors expect non-significant changes in underlying prices and create the following portfolio:

2.1. The short position of a European call option portfolio.

2.2. The short position of a portfolio of European put options of the same series.

The short cone is also known as a top straddle and generates limited gains if the underlying portfolio is priced close to settlement prices, limited losses if the underlying portfolio is priced below settlement prices, and unlimited losses if the underlying portfolio is priced above settlement prices, i.e., investors expect the underlying prices to be close to settlement prices.

Coverage combinations can also be analyzed: long strips, short strips, long straps, and short straps. As well as other hedges and strategies with option portfolios, longer hedging periods can be analyzed to apply IFRS-9 accounting.

The recommendations for investors are to perform analyses to identify, quantify, and minimize risks. Perform dynamic hedging with the financial product to be hedged (currency). Create strategies with portfolios that limit the potential losses acquired by the contracted obligations, which is equivalent to reinsurance (transferring the risk with the appropriate trading of another product, including derivatives).

Conclusions

The analyses performed on returns present stylized events. Descriptive statistics and α-stable parameters indicate that investors earn positive returns on average. The highest risk measured with the standard deviation or with the scale parameter is for the yen, the euro, the pound sterling, and the dollar, i.e., the dollar is the currency with the highest return and the lowest risk, the euro is the currency with the second best return and is the third riskiest exchange rate, the pound sterling is the currency with the third best return and is the second riskiest exchange rate, the yen is the fourth best return and the riskiest exchange rate. The returns have positive skewness coefficients, so the average returns are larger than the median and mode, and the distributions are spread more toward gains than losses. The returns have positive kurtosis coefficients, so the Gaussian distributions underestimate the relevant and null utilities but overestimate the moderate utilities.

The estimates of the α-stable parameters are consistent with the descriptive statistics because they indicate the presence of skewness and leptokurtosis, and the estimates of the confidence intervals of the parameters of the α-stable distributions confirm that returns exhibit leptokurtosis and positive skewness, so α-stable distributions more appropriately quantify profits and risks than Gaussian distributions, hypotheses that are tested with goodness-of-fit tests. The descriptive significance levels confirm that the α-stable distributions are efficient and statistically non-significant for modeling the empirical behavior of the studied returns.

Estimates of the self-similarity exponents indicate that the returns are mean reverting. The dimensions of the time series are fractional, so risk measures such as the scale parameter, value at risk (VaR), or conditional value at risk (CVaR) are themselves underestimated because they ignore the probabilities of change in trend of returns indicated by the self-similarity exponents, and which are corrected with the risk functions. The dimensions of the time series are greater than unity and the risks are greater because the time series occupies a larger area in the plane than a deterministic curve. Descriptive significance levels indicate that the model is linear and the returns are self-similar. Therefore, stochastic α-stable processes are suitable and outperform stochastic Wiener processes for the valuation of options on the studied exchange rates.

The valuation of the options indicates that Gaussian call options on the dollar and yen have a lower trading value than α-stable options because at the time of trading $\Phi(d,\alpha,\beta)-\Phi(d;\alpha,-\beta)>\Phi(d_1)-\Phi(d_2)$, therefore, Gaussian valuations are lower than α-stable ones. Gaussian call options on the euro and the pound sterling have a higher trading value than a-stable options because at the time of trading $\Phi(d,\alpha,\beta)-\Phi(d;\alpha,-\beta)<\Phi(d_1)-\Phi(d_2)$, therefore, Gaussian valuations are higher than α-stable ones. Gaussian put options on the dollar and the yen have a lower trading value than α-stable options because at the time of trading $\Phi(d,\alpha,\beta)-\Phi(d;\alpha,-\beta)>\Phi(d_1)-\Phi(d_2)$, therefore, Gaussian valuations are lower than α-stable ones. Gaussian put options on the euro and pound sterling have a higher value than α-stable options because at the time of trading $\Phi(d,\alpha,\beta)-\Phi(d;\alpha,-\beta)<\Phi(d_1)-\Phi(d_2)$, therefore, Gaussian valuations are higher than α-stable valuations.

The hedging of the α-stable portfolio is less than the hedging by the Gaussian portfolio, financial insurance with α-stable call options on the dollar and the yen is more expensive than insurance with Gaussian options, and insurance with α-stable call options on the euro and the pound sterling is less expensive than insurance with Gaussian options. Therefore, the α-stable options on the euro and sterling quantified exchange rate risk adequately because they were out-of-the-money, and the costs for exchange rate risk hedges were lower and more closely matched to empirical distributions. The α-stable options on the dollar and the α-stable options on the yen adequately quantified the exchange rate risk. Therefore, α-stable options on the dollar and the yen are riskier for the long position.

Losses with the short position of the short α-stable put portfolio are greater than the losses of the Gaussian portfolio. Losses from writing α-stable put options on the dollar and yen are less than losses from Gaussian options, and losses from α-stable call options on the euro and sterling are greater than gains from Gaussian options. Hedges written with α-stable put options on the euro and pound sterling are the ones that mean the losses for α-stable put hedges are greater than hedges with Gaussian put options. Nevertheless, if hedges with α-stable put options on the euro and pound sterling change the settlement prices, then the maximum losses for short positions are present when the settlement prices are $S = 23.62$ and $S = 26.65$, but if the settlement prices are $S \leq 21.9498$ and $S \leq 24.7034$, then the losses for the α-stable put hedges are zero, moreover, if the put options on the euro and sterling are out-of-the-money on the trade date, the respective losses are increasing until $S = 23.62$ and $S = 26.65$, and are decreasing and asymptotic to the Gaussian losses when the respective settlement prices are $S > 23.62$ and $S > 26.65$. Sensitivity analysis based on settlement prices is important for short positions because it provides insight into profits based on settlement prices and the value of the options at the time of trading. Sensitivity analysis for settlement prices on euro and sterling options indicates that Gaussian losses are greater than α-stable losses when settlement prices are 22.50 and 25.00. Therefore, the recommendation is that investors with a short position use reinsurance with financial products (currencies, options, or forward contracts) as a hedge to minimize losses on acquired obligations. Investors with a short position must identify the risks, quantify them properly, and minimize them to partially cover the obligations incurred and minimize potential losses.

Hedging strategies and combinations are also recommended to limit potential losses on obligations incurred so as to limit potential gains by creating portfolios categorized as synthetic call or put options, call or put spreads on the upside or downside, butterfly spreads, cones, strips, or straps with long or short positions according to underlying price trends.

Options on exchange rate parities are European and traded at fair value, i.e., they are exercised only on the maturity date, but investors can trade (buy or sell) in primary or secondary markets, so investors are not required to wait until the maturity date. Therefore, portfolios are dynamic according to hedging needs.

The analysis and application of non-significant adjustments are important for option valuation to properly quantify market risk as insurance does, so markets expect actuaries, economists, and financial engineers to identify, minimize, and properly quantify risks with non-significant adjustments and accountants to apply IFRS properly.

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