Determination of tax penalties according to the principles of proportionality and equity using fuzzy logic

Determinación de multas fiscales atendiendo a los principios de proporcionalidad y equidad utilizando lógica difusa

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Abstract

In this paper, a fuzzy triangular number methodology is proposed using a fuzzy controller with the MatLab © program based on the diffuse control of Mamdani. Its main contribution is to show that fuzzy logic is an alternative tool to calculate the tax penalties gradually, given the lack of proportionality and equity in Mexico's current tax framework.

Resumen

En este trabajo se propone una metodología de números triangulares borrosos mediante un controlador difuso con el programa MatLab©, basado en el control difuso de Mamdani. Su principal contribución es mostrar que la lógica difusa es una herramienta alternativa para calcular de forma gradual las multas fiscales, ante la falta de proporcionalidad y equidad del actual marco fiscal de México.

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Introduction

In tax matters, the law provides for the assumptions of tax offenses; when taxpayers, individuals or legal entities commit an act that complies precisely with the legal hypothesis, it is indispensable that the competent authority determines its existence in order to establish the level of the penalty; taking the valuation elements provided in the regulation, the basis that will serve to set a certain amount of money as a pecuniary penalty may be established (Garza, 2000).

The Federal Tax Code provides for taxes to be levied as the legal or factual situations foreseen in the law come to pass during the period in which they occur;[1] therefore, the nature of this coercion is economical and legally foreseen as a sanction to the individual, particularly to active taxpayers of a tax penalty. The legal basis of a tax penalty requires that the penalty be provided for in the law by prohibiting certain conduct. The amount must be within the parameters and amounts established by law, and above all, the authority must express the reasons, motives, circumstances, and factual and legal situations on which it is based to qualify the conduct that constitutes a penalty (Instituto de Investigaciones Jurídicas, 2000).

Thus, the objective of this work is to propose a methodology to calculate the level of penalties for offending taxpayers using fuzzy logic, using a fuzzy controller with the MatLab© program, based on Mamdani's fuzzy control, which helps to give legal certainty to taxpayers in the payment of tax penalties; and, of course, to legislators and the judiciary in modifying the current legal framework for determining the pecuniary penalties for offending taxpayers, since it does not contemplate the application of economic sanctions gradually, thus bypassing the principles of proportionality and equity contained in the Political Constitution of the United Mexican States.

Therefore, the taxation powers exercised in Mexico by the Federation and the States, through the Congress of the Union and the state congresses, must be exercised within the framework of the constitutional guarantees established in the federal constitution under Article 31, section IV. These include "Legality," in which the law defines the assumptions of tax liability, and "Proportionality" and "Equity," in the understanding that, when contributing to public expenditures, the obligors must do so proportionally and equitably, in compliance with the provisions of the laws on the matter. It is a matter of imposing a sanction or penalty in case of non-compliance since if no obligation of compliance is established, the

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[1] See Article 6 of the Federal Tax Code, first paragraph.

individual will be disinterested by not being obligated. In each case, Tax Law establishes obligations to give, to do, not to do, and to tolerate, and provides adequate means to demand due compliance from the taxpayer, including penalties (Margáin, 2007).

In short, tax penalties can be defined as monetary penalties imposed on individuals and legal entities for omissions and acts committed and typified in the tax legislation in its different spatial scopes.

Reviewing the scope of tax equity and proportionality

The Supreme Court of Justice of the Nation has reviewed several theses on the principle of "tax equity" proposed by the Federal Constitution. It has concluded that the principle requires that the taxpayers of a tax subject to the same hypothesis of causation must be in an identical situation according to the legal norm it regulates. In other words, tax provisions must treat those in the same situation equally and unequally those taxpayers who are in a different situation (Principle of tax equity, 2011). Similarly, Bolaños (2017) states that "taxation should redress social income and wealth differences. Thus, fiscal duty is an expression of social solidarity, but taxation must be based on the contributory capacity of the subject, without violating the fundamental minimums" (p.77). Nevertheless, our tax legislation emphasizes exclusively legal hypotheses related to substantiating and motivating the economic sanctions due to the aggravating circumstances caused by the taxpayers, such as recidivism—although it overlooks the conditions in order to mitigate such sanctions following the principles of equity and proportionality of the Mexican Constitution and the multiple interpretations made by the court in this regard.

On the other hand, Riofrío Martínez (2016) argues that the principle of judicial proportionality has the formal objective of determining the "proportionality of law" or "legal reasonableness." That is to say, something can be said to be fair or not fair. This principle was born to control the excesses of power that harm the rights of citizens. Because of this, it has a constitutional rank to maintain economic order and legal certainty and avoid judicial subjectivism. Sapag (2008), for example, points out that "the idea of proportionality goes back to a traditional criterion of criminal law [...], on the one hand, that penalties should be graduated in proportion to the crime" (p.170). This is to avoid an excess of power in applying German criminal law. For us, applying tax penalties in Mexico that are proportionally graduated to avoid what Riofrío Martínez (2016) calls the irrationality and dangerousness of the lack of weighting for lacking decisional objectivity.

Therefore, it is necessary to highlight in this work that several contributions show the meaning of equity and proportionality criteria that justify the proposal to use a fuzzy logic model, which contributes to applying gradual tax penalties in proportion to the infractions, taking into account the economic and

social capacity of the subjects (social income and wealth, in particular, cash flow and geographical area) as fundamental rights that for De Domingo (2007) are deemed fair.

To begin this literature study, Soberanes (2010) shows the results of a judgment whose analysis dealt with the action on the grounds of unconstitutionality 115/2008, which describes various criteria of the ministers concerning the principle of proportionality in the imposition of a traffic penalty. It shows that in the criteria of the ministers, there is no gradation to avoid the discretion of traffic officers when imposing an economic sanction. Therefore, imposing the penalty is contrary to the essential right of proportionality.

Tirado (2011) provides an overview of the treatment of the principle of proportionality in exercising the sanctioning power of the Peruvian public administration. The author concludes that, unfortunately, the country's Constitutional Court has not constructed an interpretative model on the scope of the principle of proportionality in the control of the Public Administration. Barbuta (2011) conducted a behavioral measurement of compliance with paying tax penalties. The findings suggest that taxpayer compliance factors relate to the fairness of tax rates, marital status, and exemptions granted. To summarize, the perception of fairness and proportionality affects compliance decisions.

Arias (2012) analyzed the legal foundations and implications of applying the principle of proportionality in judicial sentencing. Therefore, for this author, judges must depart from the relative laws when fundamental rights are disproportionately affected. Duncan and Sabirianova (2012) analyzed the effects of changes in the structural progressivity of the national income tax of various countries over the period 1981-2005. Their results show that progressivity reduces income inequality.

There are other contributions about "proportionality." Specifically, Becerreril (2015) researched the tax protection related to the claim for the proportionality and equity of the contributions contained in Article 31 Section IV of the Political Constitution of the United Mexican States. Its results show that, in 2007, of the 827 cases referred to the first chamber of the Supreme Court of Justice of the Nation, 553 cases were tax administrative cases; the refunds amounted to 42 billion pesos between 2002 and 2004.

Peña (2016) contrasted the hypothesis of the regressivity of the property tax applied in Ciudad Juarez, arguing inequity in the proportionality of lower-value properties since they currently pay a higher rate than those of higher value. The results show that there is a regressivity effect in its application. Swistak (2016) researched the application of tax penalties to small businesses. He argues that beyond increasing the severity of tax penalties, taxpayers should be motivated to comply without being coerced. In other words, the taxpayer's situation and the reason for non-compliance should be considered, given that their capacity requires knowledge of segmentation and personal, social, and cultural regulations.

Finally, Musayev and Musayeva (2018) propose a model using a fuzzy integral equation since there is a hidden economy with information gaps, causing a problem in calculating a tax base. Therefore,

they determine an optimal value of the tax burden of the economy proportionally increasing in a context of uncertainty.

Literature review on fuzzy logic

The theory of fuzzy sets has wide application in social sciences. Thus, this work must show some applications in which this mathematics based on multivalent logic can be a tool for analyzing imprecise information and, therefore, finding approximate data to the questions posed in a context of uncertainty. Specifically, Amozurrutia (2006) shows Fuzzy Logic and Artificial Neural Networks as two theories that enrich the analytical perspective of sociology. Their application ranges from the modeling of interaction systems and organizational systems to the simulation of agents and social phenomena.

Meanwhile, González, Muzaurieta and Espín (2008) worked on a mathematical model based on compensatory fuzzy logic and an expert system for selecting loads from abroad to detect possible economic fraud. The model was simulated using MatLab© software.

On the other hand, Rivera and Muñoz (2010) designed a methodology based on fuzzy logic applied to evaluating processes concerning state aid policies for European industry due to the difficulty of measuring these concepts. Li and Huang (2011) proposed a fuzzy programming method to develop an optimal municipal solid waste management strategy with uncertain information about Changchun City, China. The results indicate that useful solutions have been generated for planning management practices. In turn, Flores and Camarena (2013) present an innovative way to contribute to understanding social reality, highlighting examples in which fuzzy logic is applied to social knowledge to delve into the representations of fuzzy sets and their mathematical expressions.

On the other hand, Enciso, Acosta, and Campo (2013) used a fuzzy inference system with economic variables to project the annual inflation rate. Zhang, Tian, and Li (2014) developed a series of intuitionistic fuzzy trapezoidal operators for multi-attribute group decision-making. Flores and Vallejo (2015) applied fuzzy logic to evaluate the "Habitat" social program to observe, understand, and explain the complexity of social reality. Ekdi and Çıracı (2015) use a fuzzy logic model to evaluate and compare the advertising of public spaces in the city of Istanbul to understand the phenomenon's complexity. Garcia, Flores, and Venegas (2016) created a local sustainable development index using fuzzy logic in order to analyze and compare sustainable development in three specific dimensions of the environmental system at the local level.

Finally, and in this same logic, the work of González, Flores, and Gil La Fuente (2017) proposes a methodology in uncertainty management (fuzzy logic) to support decision-making in Legal Sciences.

Their work provides further elements[2] for better orientation in applying the law to assign a penalty in years of imprisonment to a particular crime. In addition, Morales and Morales (2017) designed an instrument to measure poverty. Their results reveal that fuzzy theory solves some of the methodological problems faced by poverty thresholds and marginalization indexes.

Question statement and methodology

The Federal Tax Code in force establishes that the tax authority, when imposing penalties for the commission of the infractions established in the hypotheses contained in the tax laws, in addition to being grounded and motivated, must necessarily set them within the limits of the same Code.[3] Specifically, the same law establishes the percentages and amounts that must be applied between a minimum and maximum interval, according to the legal precepts contained in the pecuniary penalties related to tax infractions and offenses.[4]

The problem is that the Federal Tax Code needs to consider criteria for the authority to base the proportional and equitable imposition of tax penalties. Moreover, the absence of a legal framework that takes greater account of constitutional principles in the application of financial penalties is one of the main factors of uncertainty since the law mentioned above specifically states the aggravating factors for the application of the maximum penalty, according to the acts committed by taxpayers, such as recidivism and the use of false documents to record non-existent transactions, including that the commission of the infraction is continuous. Nevertheless, from Sampé's (2012) point of view, equity and proportionality do not mean that the obligated subjects are equal, with similar assets and needs, since within the framework of the Federal Constitution, there is implicitly the existence of material and economic inequalities. In particular, it could be mentioned that the legal precepts in question do not consider, for example, the existence of extenuating circumstances that reflect a more appropriate situation for taxpayers. Specifically, they do not consider their economic capacity, even though the legal framework establishes limits to be obligated subjects or beneficiaries of such provisions.

Continuing with our reasoning, the tax regulations specifically point out the taxable income, the value of the assets and the number of workers as indicators to measure the economic capacity of the taxpayers.[5] However, it is noteworthy that "cash flow" is also an element to be considered because the

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[2]The elements consist of the different levels of dangerousness established for the sanctioned individuals into which the crime in question can be classified. That is, the best treatment of the uncertainty derived from the lack of data is to apply the penalty in a weighted way using fuzzy triangular numbers.
[3]See Article 75 of the 2018 Federal Tax Code.
[4]See Article 70, Title IV, Chapter I, of the 2018 Federal Tax Code.
[5]See Article 32 A of the Federal Fiscal Code

Income Tax Law[6] grants taxpayers facilities to pay taxes through a cash flow procedure. After all, it is an essential element for companies because, in this way, it indicates whether the company can pay its liabilities (Vargas, 2007; Loscalzo, 1992). It is clear here that the economic capacity of the taxpayers should be the main mitigating factor in the imposition of tax penalties. Accordingly, Sampé (2012), Barbuta (2011), Swistak (2016), Riofrío Martínez (2016), and Bolaños (2017) argue that tax provisions, in essence, should address the taxpaying capacity of the subject and legal reasonableness, including a perception of fairness so as not to influence the desire to comply.

Geographic location is another influencing factor since tax obligations are currently fulfilled through digital devices, subject to Internet connectivity, which puts taxpayers living in rural regions where this service is limited at a disadvantage. It should be added that the cost of moving suppliers is higher for companies in rural areas that are difficult to access than in urban areas. This is justifiable from the point of view of Swistak (2016) and Barbuta (2011) since the place of residence (geographic area) represents a personal situation based on knowledge of segmentation.

According to the above, a criterion that could be considered as a mitigating factor is the classification used by the National Institute of Statistics and Geography (INEGI), the Ministry of Social Development (SEDESOL), the National Council for the Evaluation of Social Development Policy (CONEVAL), and the Economic Commission for Latin America and the Caribbean (ECLAC) for establishing different degrees of urbanization considering the ranges by population size and the level of poverty by geographic zones. The specific reference points to three types of the National Urban System. The first is "Metropolitan Zones," a single unit of complete municipalities that share a central city and are highly functionally interrelated. "Conurbations" are urban formations resulting from the physical continuity between two or more geostatistical localities or urban centers, constituting a single urban unit of at least 15 thousand inhabitants. They can be inter-municipal and interstate when their population ranges between 15,000 and 49,999 inhabitants. Finally, "Urban Centers" are cities with 5,000 or more inhabitants that do not meet the characteristics of a conurbation or metropolitan area. Therefore, to determine the "degree of urbanization," a percentage of the population residing in urban areas (Metropolitan, Conurbation, and Urban Centers) should be determined compared to the total population.

In addition to the above, the per capita income by geographic area should also be considered a mitigating factor in paying for tax offenses. As a result, the concentration of businesses is more prevalent in more urbanized geographic areas (Secretaría de Desarrollo Social, 2012; López et al., 2017; Consejo

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[6]Article 196 of the Income Tax Law 2018, amended in November 2017, undoubtedly brings points of utmost relevance, such as that as of the first day of 2018, legal entities that do not exceed a limit of 5 million pesos may be taxed under the cash flow as an individual currently is.

Nacional de Evaluación de la Política de Desarrollo Social, 2007; Instituto Nacional De Estadística y Geografía, 2018; Economic Commission for Latin America and the Caribbean, 2018).

On the other hand, there are other factors to consider, especially the type of tax regime, given that there are discrepancies in complying with tax obligations. To better illustrate, consider the Régimen de Incorporación Fiscal (Tax Incorporation Regime), which is available if the income from activities in the previous fiscal year did not exceed the amount of two million pesos. Likewise, the administrative facilities granted to payment of taxes from taxpayers exclusively engaged in such activities must also be considered.[7]

This situation has forced the Supreme Court of Justice of the Nation to intervene, determining that the objective elements concerning tax equity are focused on the guarantees of equality to be considered by the legislative power to create categories or classifications of taxpayers so that they are not arbitrary and are based on objective grounds that justify a differentiated treatment between categories based, precisely, on economic and social factors. Among these, the following could be mentioned:

a) The amount of damage suffered by the treasury with the infraction (an element that is sometimes already considered in the regulation when the limits of the penalty are fixed according to the omitted tax)

b) The negligence or bad faith of the offender; or the spontaneity of their conduct in complying with the law, even if in an untimely fashion; as well as the economic capacity of the offender. The penalty must be proportional to the damage caused by the infraction, and in order to fix it, the malice and repetition of the offender must be considered. Punishment must be meted out differently to those who have different capacities so as not to hurt more those who have less, for a similar cause (MULTAS, CUANTIFICACION DE LAS, EN MATERIA FISCAL, 1976).

On the other hand, applying tax penalties needs graduation because the regulatory framework recognizes the attenuating or aggravating circumstances that would determine the penalties for infractions in this manner. Indeed, the current legislation does not contemplate a method to progressively determine an infraction, that is, the determination of a random value from a mean to a maximum penalty and from a mean to a minimum penalty. Moreover, some criteria have been issued by the courts concerning the following:

That when the amount of the penalty is fixed by the legislator for the tax omitted, giving a minimum and a maximum margin, according to that tax, in the absence of aggravating circumstances, which must be proved by the authority, or of aggravating circumstances, which must be proved by the offender, the imposition of the sanction indicated in the middle term of the penalty cannot be

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[7]See Article 111, first paragraph of the Income Tax Law.

unconstitutional in itself. Thus, for the penalty to go from medium to maximum, aggravating circumstances must be proven, and for it to go from medium to minimum, mitigating circumstances must be proven, making the offense minor, all according to the circumstances of the case and adequately reasoning the arbitration (MULTAS, CUANTIFICACION DE LAS., 1972).

Due to the above arguments, it is necessary that the tax authorities, to provide legal certainty to taxpayers in the determination of tax penalties, know a fuzzy logic methodology since the information available is imprecise, and the vagueness, in this case, forces them to propose more flexible and practical solutions to consider both aggravating and extenuating circumstances to determine the amount of the penalties. In addition, other aspects must be considered, such as economic capacity, negligence or bad faith, geographic area, type of activity, and spontaneity of the offenders' conduct.

As a result of this review, it can be observed that in recent years, there has been a notable increase in the number of academic articles proposing the use of fuzzy logic models to find solutions to social problems in general and especially economic and business problems. In our case, we highlight the works of González, Muzaurieta, and Espín (2008) and Enciso, Acosta, and Campo (2013), which use fuzzy logic models to propose solutions to problems related to decision-making in economics through a fuzzy inference system using the MatLab© program, following the type of solution we intend to develop, as well as the work of González, Flores, and Gil La Fuente (2017), which is precisely the approach intended, but oriented to the application of tax penalties.

Operation of the fuzzy inference system of the MatLab© program

The justification for the aggravating and mitigating factors expressed in this paper, specifically "Repeat offender taxpayer," "Taxpayer's cash flow," and "Taxpayer's geographic area," have their origin in the review of the literature[8] and the opinion of ten tax experts. The consultation with experts was carried out with authorities of the Association of Public Accountants of the State of Michoacán and two tax authorities of the same State in three interviews. The methodology consisted exclusively in formulating a question related to What could be the mitigating factors to weigh the payment of tax penalties by taxpayers considering the obstacles that could influence this penalty obligation. Their opinions helped to confirm the arguments following the literature and the cases presented.

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[8]In this regard, Swistak (2016) argues that taxpayers should be motivated to comply without being coerced. That is, the reasons for non-compliance must necessarily be considered, given that the capacity of taxpayers requires knowledge of segmentation and personal, social, and cultural regulations. Previously, when imposing a penalty, the sanctioning authority observed the indispensable requirements for its validity. Article 37, section I of the Federal Fiscal Code of 1967 included provisions of a personal nature. Hence, this research considers the variables expressed in the fuzzy model.

On the other hand, fuzzy control is an application that uses fuzzy expressions to formulate the rules that will intervene in such systems with its programming language. It integrates computation, visualization, and programming in a user-friendly environment where problems and solutions are expressed in the most familiar mathematical notation. This suggests a certain degree of membership of the data presented within the fuzzy sets. In this case, they will be: "Reoffending taxpayer," "Taxpayer's cash flow," and "Taxpayer's geographic area," thus enabling the diffuse controller to take different degrees of action in the system. It should be noted that the expert knowledge of one or several people is essential to devise the information based on which the decision-making process will be established.

On the other hand, fuzzy control can be applied in innumerable systems, in which they can assume values between 0 and 1, discretely or continuously, making it possible to control a complex system, such as an industrial machine or a simple domestic vacuum cleaner. This is to avoid the rigidity of classical mathematics and use a more human approach to control systems.

The structure of the operation of this most representative system using a diffuse controller is represented in Figure 1 below:
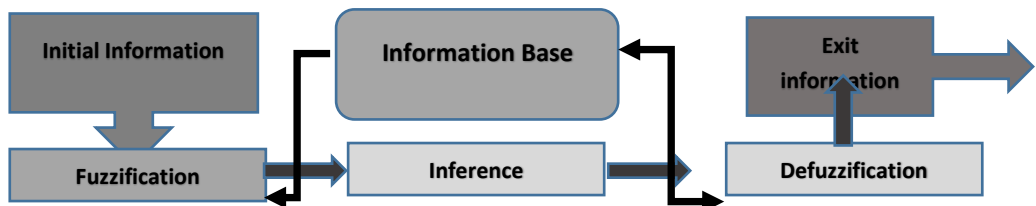

Figure 1. Structure of a fuzzy controller
Source: created by the author from Enciso, Acosta, and Campo (2013, p.75)

The schema can be explained as follows:

a)   The input data to the system will be processed to calculate the degree of membership they will have within the controller. This incorporates the knowledge of one or several subjects with the character of experts due to their vast experience in the subject under study.

b)   Secondly, the inference device and the a priori information base will be processed for the system to make decisions. Specifically, the fuzzy information system is mainly based on determining the degree of membership of the input data in the fuzzy sets (reoffending taxpayer, taxpayer cash flow, taxpayer's geographic area) of the corresponding spaces; if these are given, to decide on the output space, which in this case will be the "Amount of tax penalties."

c)   Now, the system requires the determination of rules, which are the basis of the information from the antecedent (a priori) - consequent (a posteriori), i.e., IF...THEN...In other words, it consists of devising fuzzy rules and determining the fuzzy operator using "and" or "or" according to the

relation to be found. Moreover, at this process stage, the number of rules is a function of the number of input linguistic variables and the respective fuzzy sets that form them. The fuzzy rule IF...THEN is interpreted as a fuzzy rule Q_{MM} in U×V with membership function:

$$d)\mu QMM\ (x,y) = mín\ [\mu A(x), \mu B(y)]$$

The intervention of the experts determines the type of rule due to the context in which the fuzzy inference is proposed. See Figures 2 and 3.
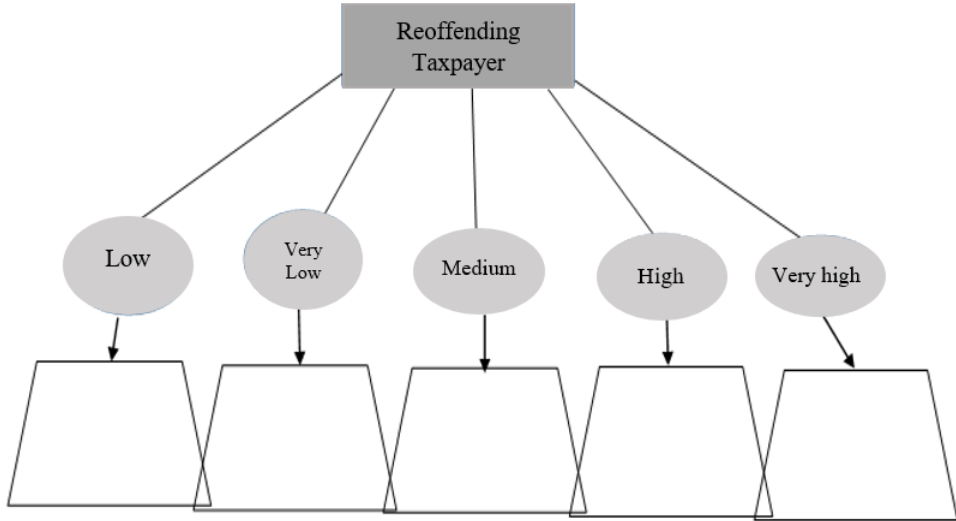

Figure 2. Structure of a fuzzy controller
Source: created by the author

Figure 2 depicts the set of semantic rules that tax law experts can use to assign different degrees of penalties to taxpayers violating the regulations.
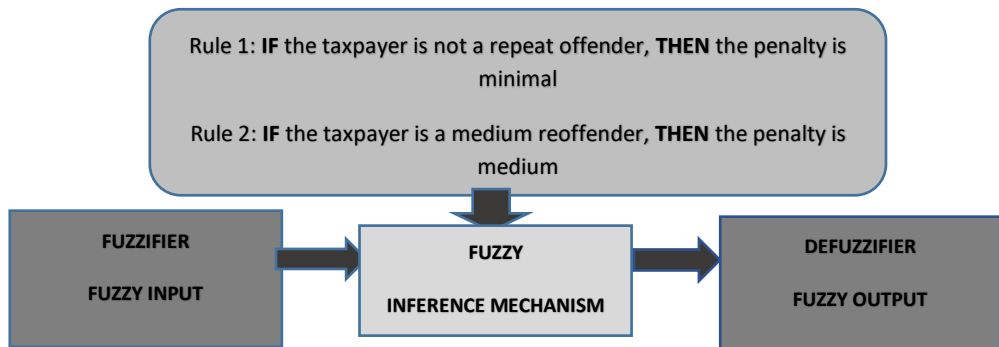

Figure 3. Fuzzy inference mechanism
Source: created by the author based on the operation of the MatLab© program

d)　Finally, in the last stage of the process, as shown in Figure 3, there is a defuzzifier inside the controller, which performs the last stage of the process to adapt the values introduced a priori to the system, providing the fuzzy values (degrees of membership) obtained from the inference in non-fuzzy values (amount of tax penalties), which are helpful for the process to be controlled. In other words, the rules are put into consideration by the implication method, obtaining, as a result, the degree of membership of the consequence of the rule in a fuzzy set in the output variable (V-sets). Subsequently, the fuzzy sets are grouped into a single set, and with the defuzzification process, the fuzzy values are converted into crisp values, in which a value representing the fuzzy relationship between the variables is obtained. Specifically, this work uses the method of the center of the area of the maximum membership, representing the most possible value.

Design of the structure of the fuzzy inference system for determining tax penalties

The proposed study can be divided into four stages: the first contains the selection of input and output variables according to the theoretical framework, the opinion of ten tax experts, and the approach proposed in this paper. The consultation with experts was carried out with authorities of the Association of Public Accountants of the State of Michoacán and two tax authorities of the same State in three interviews. The second, deals with selecting the membership functions for each linguistic assessment of the variables defined in Step 1. Although the intervention of experts at this stage of the model is crucial, it should be emphasized that since this work is a simulation, their opinions were not included. The third stage is the definition of "IF...THEN..." rules to determine the defuzzification process. Finally, the fourth stage is related to implementation and results.

The stages to be followed in the implementation of the proposed model are as follows:

- The output variables of the system will be defined based on the way in which the input variables will be structured. In other words, the outputs are directly linked by the approach of the mechanism functions. Thus, the output variable of the system is the "Amount of the Tax Penalty" because of the economic sanctions, legally established in the Federal Tax Code, imposed on active taxpayers for a tax violation. Concerning the variables for entry into the system, there is the aggravating circumstance of "Recurring taxpayer" under the same legal ordinance. Nonetheless, as proposed in this paper, two input variables should attenuate the proceeds of a tax penalty. Particularly, "Taxpayer's cash flow" is a main mitigating factor since the Income Tax Law grants facilities to taxpayers to pay taxes through a cash flow procedure; this is an essential element for the existence of companies because it indicates the capacity to pay their debts, especially short-term debts. In addition, an indicator exceeds the effectiveness of other financial statements, namely, the statement of changes in financial position, because it can be misleading

with such a format. An organization shows no change in its cash since all the surplus was invested in marketable securities. Nevertheless, this entity has significantly improved its cash flow situation despite its unchanged cash balance (Vargas, 2007; Loscalzo, 1992).

The second mitigating variable refers to the "Taxpayer's geographic area" because the degree of urbanization could be a factor that influences the level of compliance with tax penalties, according to the literature and the opinions of tax experts. Considering that the infrastructure in areas with a higher degree of urbanization includes advanced digital services, there are more roads, which decreases travel costs, both for taxpayers and suppliers, and most importantly, in such areas the per capita income is higher (Consejo Nacional de Evaluación de la Política de Desarrollo Social, 2007; López Calva et al. 2017; Secretaría de Desarrollo Social, 2012; Instituto Nacional de Estadística y Geografía, 2018; Economic Commission for Latin America and the Caribbean, 2018).

- In phase two, input and output spaces were divided into five triangular clusters. The number obeys the linguistic labels for the variables defined in Phase 1. Therefore, the partitioning of the entry spaces was carried out as follows:

For the variable "Reoffending taxpayer" (Aggravating factor), the linguistic labels are the effects of tax behavior, from the lowest to the highest degree, in terms of being Non-reoffending, Practically reoffending, Almost reoffending, Reoffending, and Completely reoffending. These are based on the criterion suggested by the experts of considering the number of occasions on which the taxpayer has been non-compliant, whether it is continuous or not, etcetera. For the variable "Taxpayer's cash flow" (Mitigating), the membership assigned was Not Solvent, Practically Solvent, Nearly Solvent, Solvent, and Fully Solvent. As mentioned above, the levels established were obtained from expert criteria and the literature. In turn, the variable "Taxpayer's geographic area" (Mitigating factor) was also graded using the labels Not urbanized, Practically urbanized, Almost urbanized, Urbanized, and Fully urbanized, considering the criteria previously mentioned. Finally, for the output variable "Amount of Tax Penalties," like the other variables, linguistic valuations were determined by assigning five membership levels. Specifically, Minimum penalty, Practically reduced, Moderately reduced, Reduced, and Fully reduced. Thus, the range of the variables was from 3,080 to 9,250, which is the economic amount, taken from the range of the penalty related to not applying for registration when obligated to do so or doing so too late [...].[9]

- In the third phase, the central part of the fuzzy controller was described, specifically the information base, based on the design of rules that will govern this inference model (IF X1 THEN X2). The terms used are therefore described, and the database is shown as a fuzzy association matrix.

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[9] See Article 79 and 80 of the 2018 Federal Tax Code.

Nevertheless, it should be noted that although a larger number of rules can be introduced to the controller, only five of them were considered for the sake of practicality in the simulation (see Table 1):

Table 1
Tax Penalty Amount Information Basis

Operators	IF	AND	AND	THEN
Rules	Reoffending taxpayer	Taxpayer's cash flow	Taxpayer's geographic area	Amount of the tax penalty
R1:	Not a Reoffender	Not Solvent	Not urbanized	Minimum penalty
R2:	Practically a Reoffender	Practically Solvent	Practically urbanized	Practically reduced penalty
R3:	Almost a Reoffender	Almost solvent	Almost urbanized	Moderately reduced penalty
R4:	Reoffender	Solvent	Urbanized	Reduced penalty
R5:	Complete Reoffender	Fully Solvent	Fully urbanized	Maximum penalty

Source: created by the author.

- Finally, the Mamdani controller method was used because this program has several toolboxes that cover various areas of technical computing, which makes it a handy instrument to enhance this type of process through the "Fuzzy Logic Toolbox." This software is very user-friendly, with a fully graphical interface, allowing users to enter the controller variables. For this case, a controller was created for the output variable "Amount of the Tax Penalty" to simulate the proposed model.

The simulation starts by entering the variables in the editor until the output variable statement is formed: "Amount of the Tax Penalty" (Figure 4)
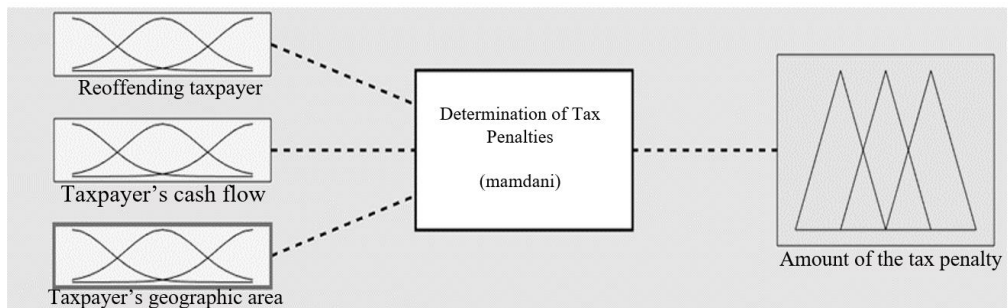

Figure 4. Declaration of the variable "Amount of the Tax Penalty"
Source: created by the author

In this same interface, the variables were defined, as established in phase two, in which the input

and output spaces were defined. That is, the ranges of the spaces and the number and type of clusters into which they were divided were defined. At the same time, the labels of each cluster were defined, as well as the parameters of each cluster, meaning the numerical points that mark the limits of each cluster (see Figures 5, 6, 7 and 8).
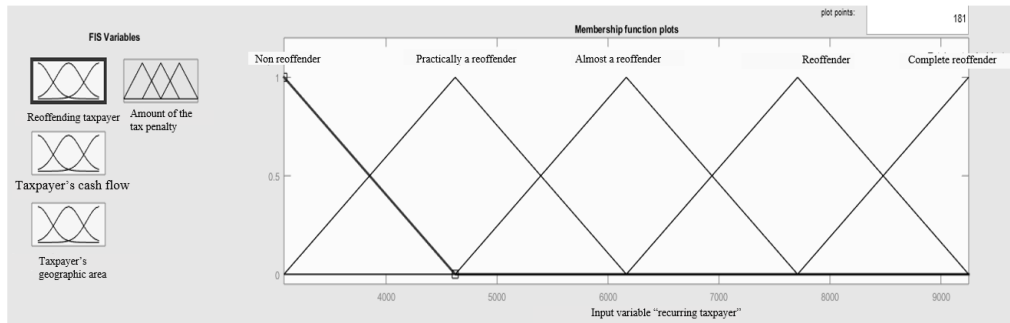

Figure 5. Input variable "recurring taxpayer"
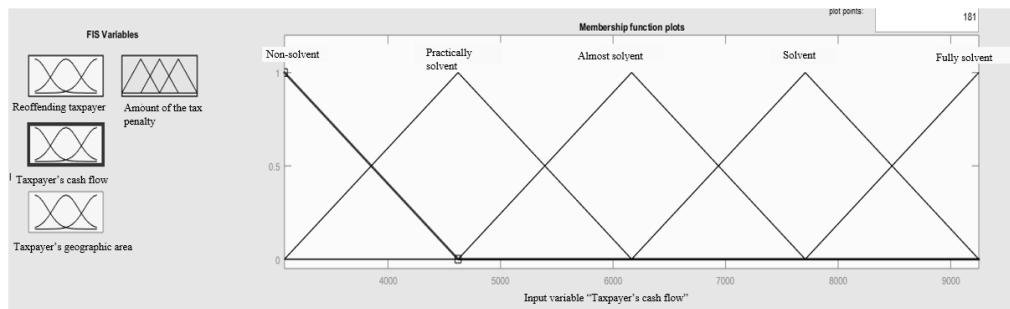Source: created by the author

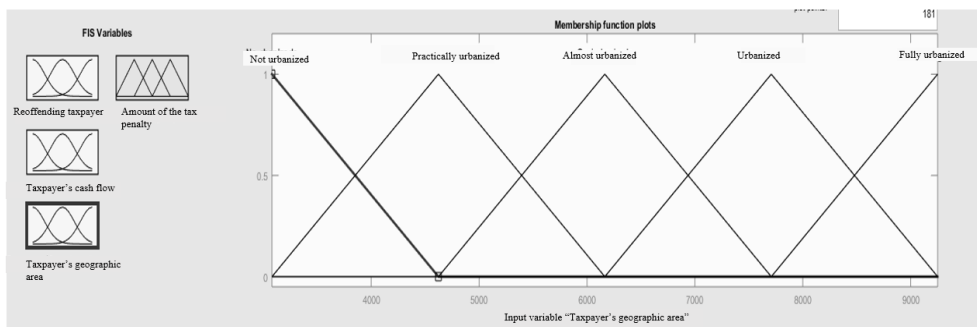Figure 6. Input variable "Taxpayer's cash flow"
Source: created by the author

Figure 7. Input variable "Taxpayer's geographic area"
Source: created by the author

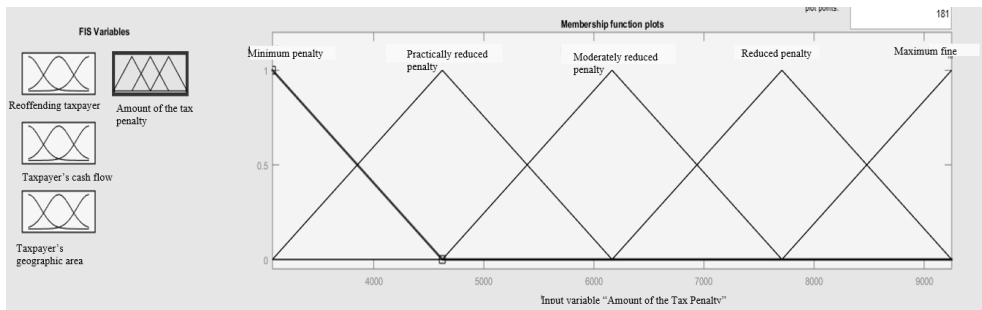Figure 8. Output variable "Amount of Tax Penalty"
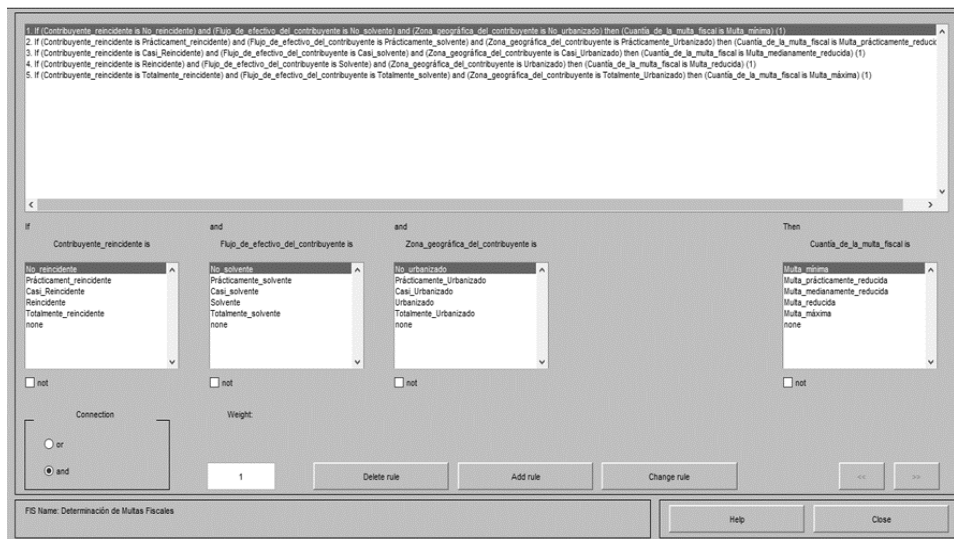Source: created by the author

Figure 9. Rules editor for the output variable "Amount of Tax Penalty"
Source: created by the author

The next step consisted of editing the rules that shape the controller to illustrate better a new interface that considers the variables already defined in the previous step and only requires selecting them together with the conditions that relate them, attached to the information base of the graphical environment, simply adding each rule (See Figure 9).

To conclude the simulation, a case study is proposed with the following information:

- Taxpayer X

- Infraction: Under Article 79 of the Federal Fiscal Code, the taxpayer did not request registration in the RFC when they were obligated to do so.

- Applicable penalty: Article 80 of the Federal Fiscal Code, Section I, establishes a

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penalty of $ 3,080 to $ 9,250.

Therefore, the program must be instructed to display the rules, which creates a new window in which the graph of the variable "recurring taxpayer" shows that the highest membership value corresponds to the polygon located in rule three, located in the far-left column. The outputs are shown in the far-right column, where the rules complied with are visible at the bottom of this column, the resulting polygon in the darker color of the inference representing the value of the amount of the tax penalty to be charged to the offending taxpayers. Figure 10 graphically shows the result of defuzzification with a line, which, in reality, is a cursor that can take any value within the variable in question and numerically at the top of the column. Particularly, in this graduation, the aggravating circumstance of "almost reoffender" is considered. Therefore, the amount of the penalty determined amounted to 6,190 pesos.
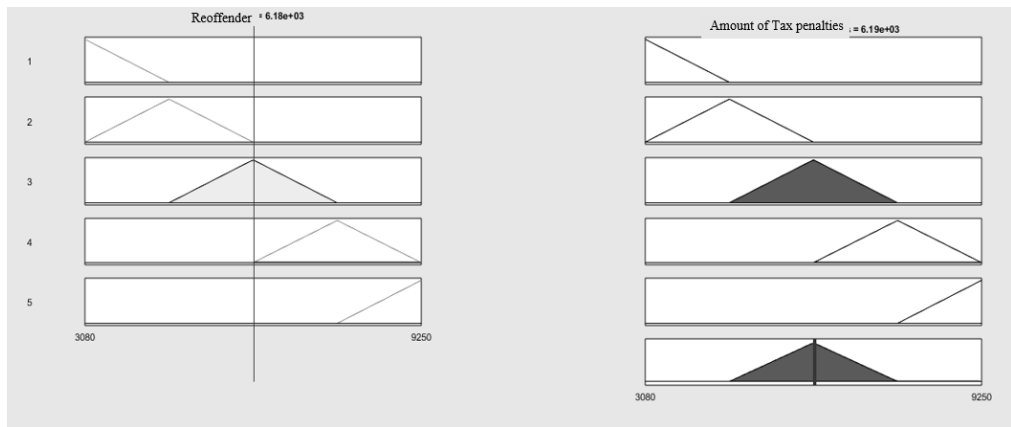

Figure 10. Output of the variable "Amount of Tax Penalties"
Source: created by the author.

Figure 11 shows another scenario for the same taxpayer. That is, a second defuzzification, where the polygon resulting from the second inference expresses on one side the values of maximum membership in rule three at the left end of the graph for the variable "Reoffending taxpayer" (Aggravating) and the same membership value at the center for the variable "Taxpayer cash flow" (Mitigating). In this case, the value of the polygon, at the extreme right in the upper part, gives an amount of 6,170 pesos corresponding to the penalty determined, considering the effect of the mitigating variable.
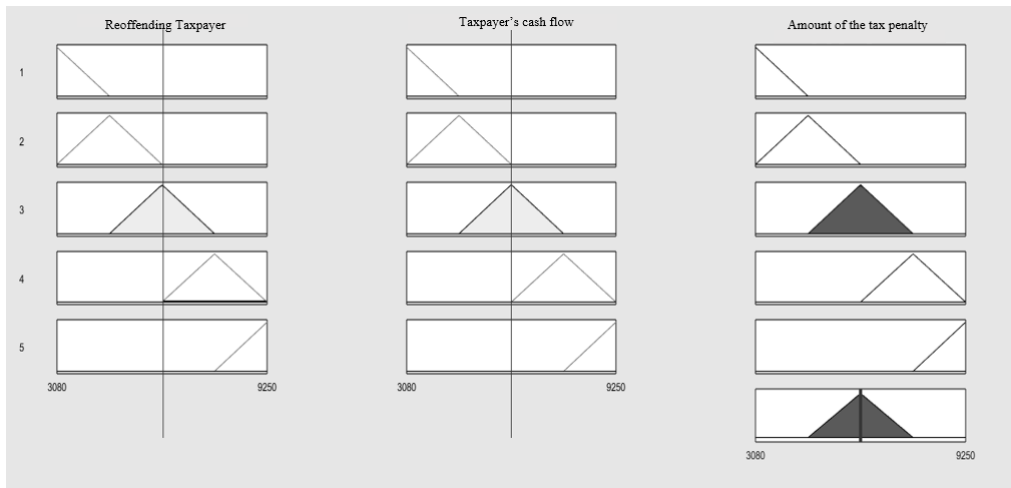
Figure 11. Output of the variable "Amount of the Tax Penalty" with only one mitigating circumstance
Source: created by the author

Now, by changing the cursor to the right in the three input variables up to rule four, they have high graduation, and therefore, the result of this new inference generates an increase in the amount of the tax penalty, as shown in the polygon on the extreme right in the lower part, and its numerical value in the upper part, by an amount equivalent to 7,700 pesos of the value of the output variable of the "Amount of the Tax Penalty" (See Figure 12).
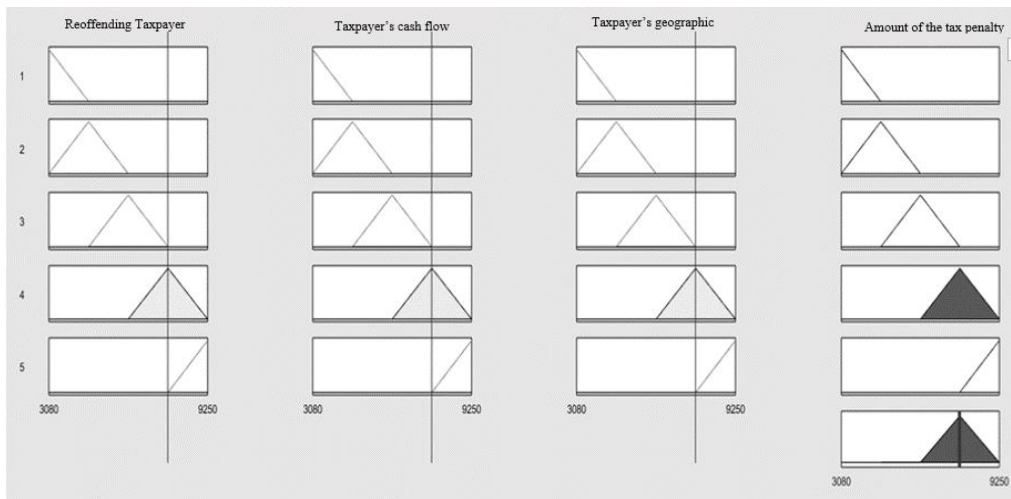

Figure 12. Output of the variable "Amount of the Tax Penalty" with the intervention of two attenuating variables with the maximum value of membership in rule four
Source: created by the author

Finally, Figure 13 shows the last assumption of the case study. This is a resulting polygon with a lower value, expressed by an amount equivalent to 3,570 pesos of tax penalty because the maximum membership values were in the first rule, which proposed membership labels for the input variables with the minimum graduation given that taxpayer X is considered: "Non-reoffending," "Non-solvent" and "Not urbanized."
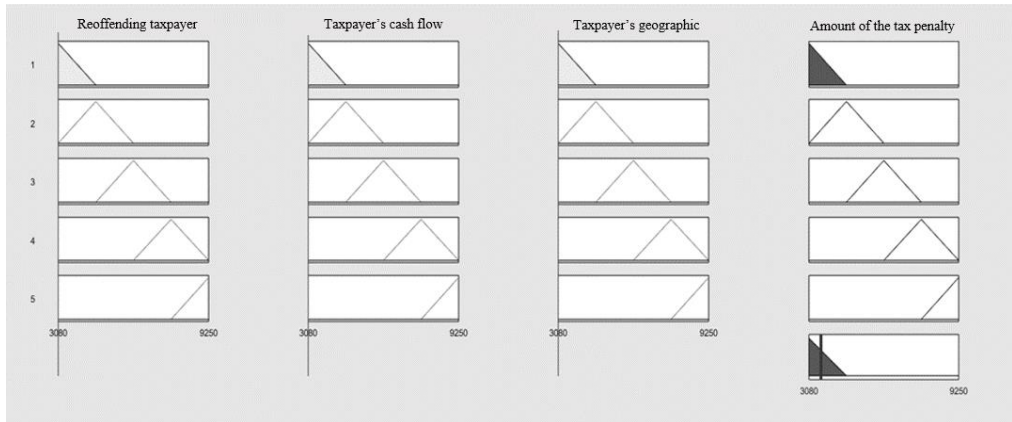

Figure 13. Output of the variable "Amount of the Tax Penalty" with the minimum value of membership
Source: created by the author

Conclusions

The imposition of tax penalties is a power that the tax authorities have by law; its proper application so far depends on the responsible parties analyzing the facts, circumstances, and omissions, as well as the commission of acts considered to aggravate the infraction, specifically, being a recurring offender or using false documentation to justify non-existent operations. This way, the authorities have the most significant number of elements to support and motivate their resolutions in accordance with the legal hypotheses contained in the Federal Tax Code. Nonetheless, the regulatory framework in the tax area needs a method for gradually determining the imposition of tax penalties. This seems to confirm that the constitutional principles of "Equity" and "Proportionality" have not been complied with up to now because the taxpayers are not recognized as having material and economic inequalities since there are no legal precepts to weigh the conditions of the taxpayers, in particular the mitigating factors.

The mitigating factors used in this work (Cash Flow and Taxpayer's geographic area) are per the fundamental principles of the Mexican Constitution in that they are objective and properly fair elements (De Domingo, 2007) focused on considering the guarantees of equality and proportionality.

Specifically, the sanctioning authority may consider the particular conditions of taxpayers, just as Swistak (2016) mentions "reasons for non-compliance" and "economic capacity," which require knowledge of segmentation. Hence, one mitigating factor analyzed is "Cash Flow."

Legal reasonableness, in this case, is the main argument put forward by the Supreme Court of Justice of the Nation, whose purpose is that the guarantees of equality must be considered by the legislative power to create categories or classifications of taxpayers because they are not arbitrary and are based on objective grounds that justify a differentiated treatment between categories based, precisely, on economic and social factors. Suppose a criterion of progressivity is applied for calculating tax penalties through a fuzzy procedure, which considers the mitigating factors that express the economic capacity and the conditions of non-compliance, the sanctioned individuals may perceive that the sanctions are equitable and proportional, which is in line with Barbuta (2011), who states that the factors of non-compliance in the payment of tax penalties are related to the fairness of the tax rates. In that case, the more fairness and proportionality taxpayers perceive, the more tax collection will increase. Moreover, there will be a contribution to the rationality and decisional objectivity of those responsible for weighting infractions (Riofrío Martínez, 2016).

On the other hand, the imposition of penalties is a discretionary power that has prevailed until now. According to the regulations on the matter, the tax authorities' judgment only determines the minimum and maximum amounts of the range established for collecting the infractions and the reductions indicated in some instances.

The contribution of this paper is to present a methodology based on fuzzy logic as an alternative tool to gradually approach the determination of tax penalties due to the lack of proportionality and equity that has prevailed until today. Legislators and the executive branch are responsible for proposing initiatives to improve how tax penalties are applied. The courts will have to do their part through the legal mechanisms available to them to improve the interpretation of this approach.

On the other hand, the fuzzy inference model shows various criteria based on subjective opinions since the information available is uncertain because there is no precise data to determine the proportion of the penalty. A different legal framework is essential so that experts, specifically the authorities in charge of determining sanctions, have more information to support and motivate the assignment of valuations. The implementation of the system consisted of selecting input and output variables. Likewise, rules were used since the fuzzy information system is mainly based on determining different membership levels of the input data in the fuzzy sets of the corresponding spaces. These rules decide the output space, such as obtaining the "Amount of tax penalties."

To continue working on this issue, it is necessary to continue researching the inclusion of other mitigating factors to achieve the most significant possible fairness and proportionality in determining tax

penalties. Specifically, one could work with the type of tax regimes, taxpayer category, taxable income, value of assets, and number of workers.

Finally, the model developed in this work shows that the fuzzy logic theory can become standard in legal areas because it can qualify the values analyzed by imitating the thinking of human beings, a process in which there may be more than one solution to legal, financial, economic, and social problems.

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